Motion Control in a Disturbed Environment

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Abstract. This paper explores the unique design characteristics of automatic control systems in the presence of environmental disturbances. Specifically, the study focuses on controlling the movement of a marine vehicle along a predetermined course, despite unknown external events. Emphasis is placed on attaining the necessary closed-loop system attributes, such as the required stability margins. Furthermore, this study examines the means by which the marine object is stabilized via control, taking into consideration external forces, potential disruptions and additional stability margins.

Keywords: Control, Modeling, Autonomous.

1 Introduction

Automatic motion control systems are widespread in the modern world. They can be found in cars, multi-purpose drones, airplanes, factory robots and ships. These systems employ advanced sensor technologies, ensuring precise and efficient control over moving objects. Consequently, there is a constant need for the development and enhancement of these systems. Although numerous control design variants for different situations are available in the literature [1-15], not all have been covered.

This article is concerned with the peculiarities of designing automatic control systems in the presence of external disturbances. Specifically, it explores how to control the movement of a marine object along a given course in the presence of unknown external influences. The focus is on achieving the necessary attributes of a closed-loop system, specifically, attaining the required degree of stability. Furthermore, the study examines the issue of stabilizing a marine surface object via formed control, while also taking into account the effects of external forces and non-core requirements on the stability characteristics of the overall system.

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2 Problem Statement and Solution Methods

The marine object under consideration is modelled mathematically using differential equations in the following form:

$$m_x \frac{dV_x}{dt} + mh\frac{d\omega_z}{dt} = G_x,$$

$$m_y \frac{dV_y}{dt} + \lambda_{26}\frac{d\omega_z}{dt} = G_y,$$

$$m_z \frac{dV_z}{dt} + (\lambda_{34} - mh)\frac{d\omega_x}{dt} + \lambda_{35}\frac{d\omega_y}{dt} = G_z,$$

$$J_x \frac{d\omega_x}{dt} + (\lambda_{34} - mh)\frac{dV_x}{dt} = T_x,$$

$$J_y \frac{d\omega_y}{dt} + \lambda_{35}\frac{dV_z}{dt} = T_y,$$

$$J_z \frac{d\omega_z}{dt} + \lambda_{26}\frac{dV_y}{dt} + mh\frac{dV_x}{dt} = T_z.$$

As an application example, consider a marine object with a maximum speed of $V = 15м/с$ and a capacity of 4500 m3.

We are going to extract from the mathematical model only the part regarding the process of movement along the course. We acquire

$$\dot{V}_z = \frac{J_{xx}(1+k_{44})G_z - mk_{34}T_x}{mJ_{xx}(1+k_{33})(1+k_{44}) - m^2k_{34}^2},$$

$$\dot{\omega}_y = \frac{T_y}{J_{yy}(1+k_{55})}.$$

Note that when studying the motion of marine vessels, the down angle ψ is considered to be small. This leaves only one equation from the kinematic equations

$$\dot{\varphi} = \omega_y.$$

Where the forces and moments G_z, T_x, T_y are expressed as below

$$G_z = Z_H + Z_R + F_z,$$

$$T_x = -mgh_0 + z_k mV_x\omega_y + M_{xH} + M_{xR} + M_x,$$

$$T_y = M_{yH} + M_{yR} + M_y.$$

Here Z_H, M_{xH}, M_{yH} – hydrodynamic force and moment projections that impact the steerable object, Z_R, M_{xR}, M_{yR} – projections of force and moment occurring when changing the angle of vertical control rudders, F_z, M_x, M_y – projections of force and moment vectors from external forces in the coordinate system $Oxyz$.

To obtain these projections, the corresponding formulas are used as follows

$$Z_H = 4{,}89V_L^2\beta + 9{,}644V_L^2\Omega\sqrt{1-\Omega^2} + 23{,}7V_L^2\beta|\beta|,$$
$$M_{xH} = -11{,}39V_L^2\beta - 22{,}474V_L^2\Omega\sqrt{1-\Omega^2} - 55{,}23V_L^2\beta|\beta|,$$
$$M_{yH} = -322{,}61V_L^2\beta - 223{,}6V_L^2\Omega + 69{,}1V_L^2\Omega|\beta| - 160{,}8V_L^2\Omega|\Omega|,$$

where $V = \sqrt{V_x^2 + V_y^2}$, $V_L = \sqrt{V^2 + \omega_y^2 L^2}$, $\Omega = \dfrac{\omega_y L}{V_L}$, $\beta = -\mathrm{arctg}\left(\dfrac{V_z}{V_x}\right)$.

$$Z_R = 1{,}236V_{LR}^2\beta - 0{,}567V_{LR}^2\omega - 1{,}236V_{LR}^2\delta,$$
$$M_{xR} = -3{,}91V_{LR}^2\beta + 1{,}79V_{LR}^2\omega + 3{,}91V_{LR}^2\delta,$$
$$M_{yR} = 156{,}93V_{LR}^2\beta - 29{,}9V_{LR}^2\omega - 156{,}93V_{LR}^2\delta,$$
$$V_{LR} = \sqrt{V^2 + L_R^2\omega_y^2},\ \omega = \frac{\omega_y L}{V_{LR}}.$$

In the above formulas δ – angle of vertical control rudders.
To investigate the dynamics of a marine object, the thruster equation must be appended

$$\dot{\delta} = u,$$

where u – desired function of the control action. The vertical rudder control angle is also limited by technical constraints. These must also be taken into consideration: $\max|\delta| = 30°$, $\max|u| = 3°/\sec$.

To develop the control we need to linearize the mathematical model in the vicinity of zero on the variables φ, ω_y, V_z. Then at a fixed velocity $V_x = V$ we achieve a mathematical model of course motion in linear approximation

$$\dot{V}_z = -a_{11}VV_z + a_{12}V\omega_y - b_1V^2\delta + d_1F_z,$$
$$\dot{\omega}_y = a_{21}VV_z - a_{22}V\omega_y - b_2V^2\delta + d_2M_y,$$
$$\dot{\varphi} = \omega_{y.}$$

For the marine vessel under consideration in the example, we have prescribed values of the constant coefficients:

$$a_{11} = 8{,}3763\cdot10^{-3},\quad a_{21} = 2{,}5823\cdot10^{-4},$$
$$a_{12} = 1{,}6228,\qquad a_{22} = 0{,}052989,$$
$$b_1 = 1{,}7038\cdot10^{-3},\quad b_2 = 2{,}4459\cdot10^{-4},$$
$$d_1 = 1{,}3255\cdot10^{-3},\quad d_2 = 1{,}5586\cdot10^{-6}.$$

In the process of researching the motion of marine vessels, as a rule, it is accepted the drift as one of the parameters of the state vector β as opposed to the V_z velocity projection. Considering β and V_z in the linear approximation. They are related as follows $\beta = -\dfrac{V_z}{V_x} = -\dfrac{V_z}{V}$, so $V_z = -V\beta$.

With this equation, we can define a linearised mathematical model of the dynamics of a moving object in the horizontal plane.

$$-V\dot{\beta} = a_{11}V^2\beta + a_{12}V\omega_y - b_1V^2\delta + d_1F_z,$$
$$\dot{\omega}_y = -a_{21}V^2\beta - a_{22}V\omega_y - b_2V^2\delta + d_2M_y, \Rightarrow$$
$$\dot{\varphi} = \omega_{y.}$$

$$\dot{\beta} = -a_{11}V\beta - a_{12}\omega_y + b_1V\delta - \frac{d_1}{V}F_z,$$
$$\Rightarrow \dot{\omega}_y = -a_{21}V^2\beta - a_{22}V\omega_y - b_2V^2\delta + d_2M_y,$$
$$\dot{\varphi} = \omega_{y.}$$

If as a state vector we consider $x = (\beta \quad \omega_y \quad \varphi)'$ and as a vector of external forces $w = (F_z \quad M_y \quad 0)$, then we can use a new notation

$$\tilde{a}_{11} = -a_{11}V,\ \tilde{a}_{12} = -a_{12},\ \tilde{b}_1 = b_1V,\ \tilde{d}_1 = -\frac{d_1}{V},$$
$$\tilde{a}_{21} = -a_{21}V^2,\ \tilde{a}_{22} = -a_{22}V,\ \tilde{b}_2 = -b_2V^2,\ \tilde{d}_2 = d_2,$$

and get a linear system of differential equations describing the motion of the examined vessel in the horizontal plane in matrix form

$$\dot{x} = Ax + B\delta + Dw$$

with matrices

$$A = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & 0 \\ \tilde{a}_{21} & \tilde{a}_{22} & 0 \\ 0 & 1 & 0 \end{pmatrix},\ B = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ 0 \end{pmatrix},\ D = \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ 0 \end{pmatrix}.$$

Now we need to take into account that the effects of external forces $w(t)$ are not completely determined, but obey the constraint

$$\|w(t)\|_\infty \le 1 \text{ when } 0 \le t < \infty .$$

We will consider jointly the equations of motion and the vertical rudders equation, and introduce an extended state vector $\tilde{x} = \begin{pmatrix} x \\ \hline \delta \end{pmatrix}$. Then the extended system of linear differential equations will appear as follows

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u + \tilde{D}\tilde{w},$$

where the new matrices have the form

$$\tilde{A} = \left(\begin{array}{c|c} A & B \\ \hline 0_{1x3} & 0 \end{array}\right),\ \tilde{B} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},\ \tilde{D} = \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ 0 \\ 0 \end{pmatrix},\ \tilde{w} = (F_z \quad M_y \quad 0 \quad 0).$$

We use control in the form of static state feedback

$$u = K_x x + K_\delta \delta = K\tilde{x} = k_1\beta + k_2\omega + k_3\varphi + k_4\delta .$$

Because there is uncertainty in the definition of the disturbing forces $\tilde{w}$ let us consider the best dynamics of our system given this uncertainty. So k_1, k_2, k_3, k_4 are to be determined in the process of consideration of minimisation of the size of the J_d invariant ellipsoid to achieve the necessary characteristics of our system

$$J_d = J_d(K) \to \min_{K \in \Omega_{sk} \subseteq \Omega_k} .$$

Assume that we measure the entire vector $\tilde{x}$,

$$y = C\tilde{x},\ \ C = E_{4\times 4}.$$

Then the equations of the closed system are as follows

$$\dot{\beta} = \tilde{a}_{11}\beta + \tilde{a}_{12}\omega_y + \tilde{b}_1\delta + \tilde{d}_1 F_z,$$

$$\dot{\omega}_y = \tilde{a}_{21}\beta + \tilde{a}_{22}\omega_y + \tilde{b}_2\delta + \tilde{d}_2 M_y,$$

$$\dot{\varphi} = \omega,$$

$$\dot{\delta} = k_1\beta + k_2\omega + k_3\varphi + k_4\delta,$$

or in matrix form

$$\dot{\tilde{x}} = A_3\tilde{x} + D_3\tilde{w},\ \ A_3 = \left(\begin{array}{ccc|c} & A & & B \\ \hline (k_1 & k_2 & k_3) & k_4 \end{array}\right),\ \ D_3 = \tilde{D} = \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ 0 \\ 0 \end{pmatrix}.$$

The desired vector of regulator coefficients is sought, which compensates the limited external disturbing influences in the best possible way (in the sense of the problem at hand) and at the same time provides the desired degree of stability of the closed-loop system:

$$k_1 = 3.0981,\ \ k_2 = 57.7472,\ \ k_3 = 23.9121,\ \ k_4 = -1.2793.$$

Herewith we have the following components of the vector γ, that achieves the desired degree of stability:

$$\gamma_{11} = -1,\ \ \gamma_{12} = 2,\ \ \gamma_{21} = 1,\ \ \gamma_{22} = -0.05.$$

Using the implemented controller, the eigenvalues of the matrix of the closed-loop system become as follows

$$\lambda_1 = -0.1027,\ \ \lambda_2 = -0.5465 + 0.0147i,$$

$$\lambda_3 = -0.5465 - 0.0147i,\ \ \lambda_4 = -0.5368,$$

i.e. the desired degree of stability has been achieved.

We note that consideration of the requirement for the stability characteristic is very important in solving practical cases. Let us now compare the result obtained here with the control that compensates for limited perturbations without additional modal requirements. For this controller we obtain the coefficients:

$$\tilde{k}_1 = 0.1738,\ \ \tilde{k}_2 = 10.6013,\ \ \tilde{k}_3 = 1.0242,\ \ \tilde{k}_4 = -0.0378.$$

Then the eigenvalues of the matrix of the considered system are equal to

$$\tilde{\lambda}_1 = -0.3138,\ \tilde{\lambda}_2 = -0.0599 + 0.1886i,$$
$$\tilde{\lambda}_3 = -0.0599 - 0.1886i,\ \tilde{\lambda}_4 = -0.0574,$$

i.e. the degree of stability when using the second controller is estimated by the constant 0.06, which is much worse than the stability characteristic of the system with the first controller and shows itself poorly in the dynamics of the object and increases the duration of the rotation process.

We can now compare the behaviour of the dynamical system in different conditions. Assume the bounded external forcing is a sequence of random bounded "bursts" of 40 seconds duration.

In Fig. 1 and Fig. 2, the solid line represents the vessel dynamics (yaw change and rudder deviation, respectively) when using the first controller, while the dashed line represents the same processes when applying the second controller, constructed without taking into account the requirement for the degree of stability.

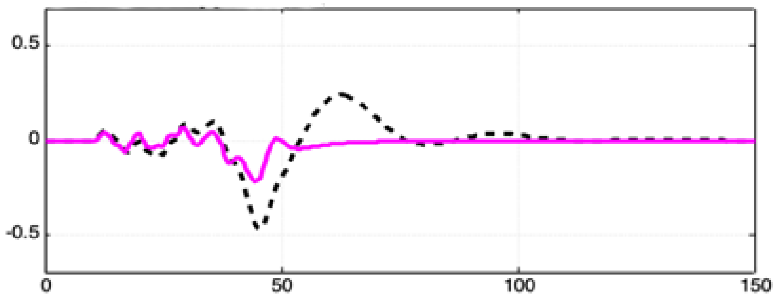

Fig.1. Vessel's yaw change.

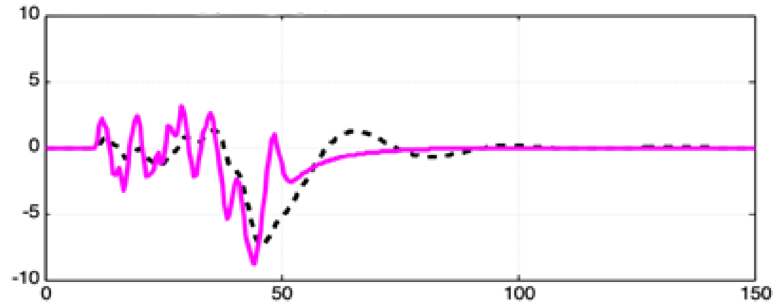

Fig. 2. Deviation of the rudders.

According to Fig. 1, when using the first control, the ship's deviation from the course is 5 times less than for the second control. At the same time, the time to stabilise the ship's course after the end of the limited disturbance is less than 10 seconds in the first case, and 50 seconds in the second case.

3 Conclusion

The article presents and considers the problem of controlling the motion of a ship in the horizontal plane in the presence of uncertain external disturbances. A special place is given to the achievement of the necessary characteristics of the system, in particular, the achievement of the necessary stability characteristic. Computer modelling is performed in MATLAB-Simulink.

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