Computer Simulation of the Elastic Regime of the Development of a Single-Well Reservoir with an Unknown Initial State

Khanlar Gamzaev[0000-0002-1228-7892] and Sevinc Kerimova
Azerbaijan State Oil and Industry University
Baku, Azerbaijan
gamzaevkhanlar@gmail.com
kerimovasevinc66@gmail.com

Abstract. To describe the elastic regime of the development of a single-well oil reservoir, a model of a turbulent filtration flow of a single-phase liquid is proposed, presented in the form of a nonlinear parabolic equation with respect to the volume flow of the liquid. The laws of change in the time of the well flow rate and the pressure at the bottom of the well are considered to be set. The distribution of fluid flow in the reservoir at the initial moment of time, as well as the presence of a hydrodynamic connection between the reservoir and the external region, are considered unknown. Within the framework of the proposed model, the task is to determine the distribution of fluid flow in the reservoir, including the boundary. This problem belongs to the class of boundary value inverse problems without initial conditions. To numerically solve the inverse problem, a discrete analogue of the problem on a uniform difference grid is first constructed using the method of difference approximation. To solve the resulting system of linear equations, a computational algorithm is proposed that makes it possible to determine the flow rate of a liquid at all nodal points of the calculated part, with the exception of a certain triangular area. The calculated value of the fluid flow rate at the outer boundary of the formation determines the presence or absence of a hydrodynamic connection between the formation and the outer region. Numerical experiments for a model oil reservoir were carried out based on the proposed computational algorithm.

Keywords: single-well oil reservoir, elastic development mode, turbulent filtration law, boundary value inverse problem without initial conditions, difference approximation method.

1 Introduction

It is known that in the initial period of oil field development, reservoir pressure significantly exceeds the pressure of oil saturation with gas and a natural elastic development regime occurs in the reservoir. In the elastic mode of development, oil inflow to wells occurs due to the use of energy from the elastic expansion of the oil itself and the formation rock. Currently, a mathematical model of the unsteady filtration flow of a single-phase liquid in a reservoir is mainly used to analyze the elastic regime of an oil reservoir and forecast development indicators. This model includes the differential equation of continuity of a single-phase filtration flow, the equation of fluid flow motion, which uses one or another empirical filtration law, and additional equations of state for a liquid and a porous medium [1-3]. In practice, when modeling a single-phase filtration flow in a reservoir, Darcy's law is mainly used as the equation of motion, expressing the linear dependence of the filtration rate on the pressure gradient. However, Darcy's law applies only to Newtonian fluids in a certain limited range of filtration rates, in which turbulence, inertia, and other high-velocity effects are negligible. Numerous experimental studies show that as the filtration rate increases, first due to inertial effects, and then due to turbulence in the fluid flow, a deviation from Darcy's law occurs. It is believed that this range of filtration rates can be adequately represented by the so-called law of turbulent filtration, which describes both laminar flow and flow with inertial-turbulent effects in the reservoir [2–4].

Obviously, in order to solve practical problems related to the development of reservoirs in an elastic regime, in addition to the mathematical model, it is necessary to know the initial and boundary conditions describing the initial state of the reservoir and the interaction of the reservoir with its surrounding area. However, it should be noted that access to the oil reservoir is limited and is possible only through wells opening the reservoir in small areas, the initial pressure distribution in the reservoir, as well as the pressure and flow of oil at the outer boundary of the reservoir are not available for direct measurements. Therefore, it is practically impossible to accurately represent the initial conditions in the formation and the conditions at the outer boundary of the formation. The main sources of information about the processes occurring in the reservoir during development are production wells, where the pressure and flow rate of the well are available for direct measurements. In this regard, for the practice of developing oil reservoirs in an elastic regime, it is important to determine development indicators based only on information obtained from wells.

In this paper, we study the issue of numerical modeling of the elastic regime of the development of a single-well reservoir using the law of turbulent filtration, in the absence of an initial condition and a condition at the outer boundary of the reservoir.

2 Problem statement and solution method

Let us consider a homogeneous cylindrical oil-bearing reservoir with a thickness h and radius R , bounded from above and below by impenetrable planes. In the center of the cylindrical reservoir there is a hydro dynamically perfect borehole of radius r_W, the axis of which coincides with the axis of the cylinder. The pressure at the bottom of the well exceeds the pressure of oil saturation with gas and the reservoir is developed in an elastic mode. It is assumed that the inflow of oil from the reservoir to the well is a plane-radial flow of a single-phase liquid obeying the law of turbulent filtration. Considering oil to be a weakly compressible viscous liquid, we will present a mathematical model of the elastic regime of the development of this reservoir in the form of a system of equations that includes the differential equation of continuity of a single-phase filtration flow

$$\beta^* \frac{\partial P(r,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(ru(r,t)\right), \tag{1}$$

and the equations of motion of the filtration flow, which uses the law of turbulent filtration

$$\frac{\partial P(r,t)}{\partial r} = \frac{\mu}{k}u(r,t) + \beta\rho u^2(r,t), \tag{2}$$

where $P(r,t)$ is the pressure, $u(r,t)$ is the filtration rate of the liquid (oil), ρ is the density of the liquid, μ is the dynamic viscosity of the liquid, k is the absolute permeability of the porous medium, β^* is the elastic capacity coefficient of the reservoir, β is the turbulence coefficient.

Taking into account the relationship between the filtration rate $u(r,t)$ and the volumetric flow rate of liquid $Q(r,t)$, we represent the systems of equations (1) and (2) in the form

$$\frac{\partial P(r,t)}{\partial t} = \frac{1}{2\pi rh\beta^*}\frac{\partial Q(r,t)}{\partial r}, \tag{3}$$

$$\frac{\partial P(r,t)}{\partial r} = \frac{\mu}{2\pi rhk}Q(r,t) + \frac{1}{(2\pi rh)^2}\beta\rho Q^2(r,t). \tag{4}$$

The systems of equations (3) and (4) can be reduced to a single equation with respect to the volume flow of a liquid $Q(r,t)$. To do this, we differentiate both parts of equation (4) by the variable t

$$\frac{\partial^2 P(r,t)}{\partial r\partial t} = \left(\frac{\mu}{2\pi rhk} + \frac{\beta\rho}{2(\pi rh)^2}Q(r,t)\right)\frac{\partial Q(r,t)}{\partial t}.$$

Substituting the expression $\frac{\partial P(r,t)}{\partial t}$ from equation (3) here, we obtain a nonlinear parabolic equation with respect to the function $Q(r,t)$

$$\left(\frac{\mu\beta^*}{k} + \frac{1}{\pi rh}\beta\beta^*\rho Q(r,t)\right)\frac{\partial Q(r,t)}{\partial t} = \frac{\partial^2 Q(r,t)}{\partial r^2} - \frac{1}{r}\frac{\partial Q(r,t)}{\partial r}. \tag{5}$$

Obviously, in order to determine the distribution of the volume flow of a liquid in the elastic mode of development based on model (3), it is necessary to have initial and boundary conditions describing the initial state of the reservoir and its interaction with the surrounding area.

Since the flow rate of the well and the pressure at the bottom of the well are available for direct measurements, the boundary conditions at the well can be represented as

$$Q(r_W,t) = Q_W(t), \tag{6}$$

$$P(r_W,t) = f(\,t\,), \tag{7}$$

where $Q_W(t)$ and $f(t)$ are the specified functions.

However, due to the fact that the initial distribution of fluid flow in the formation and the fluid flow at the outer boundary of the formation $r = R$ are not directly measurable, it is not possible to formulate the initial condition and boundary condition corresponding to the interaction of the formation with its surrounding area. Using the equation of continuity of the filtration flow (3) and the boundary condition with respect to pressure (7), we obtain an additional condition with respect to fluid flow in the well

$$\frac{\partial Q(r_w,t)}{\partial r} = 2\pi r_w h \beta^* \frac{df(t)}{dt}. \tag{8}$$

Thus, the task of determining the dynamics of the distribution of the volume flow of liquid in the reservoir under consideration is reduced to solving equation (5) under conditions (6), (8). The assigned task (5), (6), (8) it belongs to the class of boundary-value inverse problems without initial conditions [5-8]. Numerous works have been devoted to theoretical research and the development of numerical methods for solving inverse problems related to the restoration of boundary conditions [5, 6, 9-18].

For the numerical solution of the inverse problem (5), (6), (8), first, we discretize a given rectangular area $\{r_w \le r \le R,\ \ 0 < t \le T\}$ in space and time, with steps $\Delta r = \frac{R - r_w}{n}$ in the variable r and $\Delta t = \frac{T}{m}$ in the variable t

$$\overline{\omega} = \{(r_i, t_j) : r_i = r_w + i\Delta r,\ t_j = j\Delta t,\ i = 0,1,2,...n,\ j = 0,1,2,...m\}.$$

Let's build a discrete model of the problem (5), (6), (8) on a grid $\overline{\omega}$ using the difference approximation method. Using an explicit time approximation for the nonlinear coefficient in equation (5), we present a discrete model of the problem as

$$\left(\frac{\mu\beta^*}{k} + \frac{\rho\beta\beta^*}{\pi r_i h} Q_i^{j-1}\right)\frac{Q_i^j - Q_i^{j-1}}{\Delta t} = \frac{Q_{i+1}^j - 2Q_i^j + Q_{i-1}^j}{\Delta r^2} - \frac{1}{r_i}\frac{Q_i^j - Q_{i-1}^j}{\Delta r}, \tag{9}$$

$$i = \overline{1, n-1}, \qquad j = \overline{1, m},$$

$$Q_0^j = Q_w{}^j, \qquad j = \overline{0, m}, \tag{10}$$

$$\frac{Q_1^j - Q_0^j}{\Delta r} = 2\pi r_w h \beta^* \frac{f^j - f^{j-1}}{\Delta t},\ j = \overline{0, m}, \tag{11}$$

where $Q_i^j \approx Q(x_i, t_j)$, $f^j = f(t_j)$, $Q_w{}^j = Q_w(t_j)$.

As you can see, the discrete model of the problem (5), (6), (8) for each fixed value j, $j = 1, .2\,...\,, m$, it is a system of linear difference equations in which the unknowns Q_i^j, $i = 1,2,...n,$ are the approximate values of the desired function $Q(r,t)$ at the nodal points of the difference grid $\overline{\omega}$.

According to conditions (10), (11) on two spatial layers $r_0 = r_w$, $r_1 = r_0 + \Delta r$ the values of the variables Q_0^j and Q_1^j are considered to be set (in the figure, the corresponding nodes are indicated by circles), i.e.

$$Q_0^j = Q_w^j, \qquad Q_1^j = Q_0^j + 2\pi r_w h \beta^* \Delta r \frac{f^j - f^{j-1}}{\Delta t}.$$

Now the task is to find from the system of equations (9) the approximate values of the desired function $Q(r,t)$ at the remaining nodal points, i.e. Q_i^j, $i = \overline{2,n}$, $j = \overline{0, m}$.

A simple analysis of the system of equations (9) shows that from this system it is consistently possible to determine:

Q_2^j, for $i = 1$, $j = \overline{1,m}$;

Q_3^j, for $i = 2$, $j = \overline{2,m}$;

Q_4^j, for $i = 3$, $j = \overline{3,m}$;

Q_5^j, for $i = 4$, $j = \overline{4,m}$;

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Q_n^j, for $i = n-1$, $j = \overline{n-1,m}$.

In the figure, the grid nodes corresponding to the calculated variables are indicated by asterisks. However, as shown in the figure, in some nodes located in the triangular region D, the values of the desired function are not determined.

The numerical implementation of the above calculation procedure when setting Q_0^j and Q_1^j can be carried out using the recurrent formula

$$Q_{i+1}^j = \Delta r^2 \left(\frac{\mu\beta^*}{k} + \frac{1}{\pi r_i h}\beta\beta^* \rho Q_i^{j-1}\right)\frac{Q_i^j - Q_i^{j-1}}{\Delta t} + \frac{\Delta r\left(Q_i^j - Q_{i-1}^j\right)}{r_i} + 2Q_i^j - Q_{i-1}^j, \qquad (12)$$

$$i = \overline{1, n-1}, \quad j = \overline{i, m}.$$

Thus, formula (12) allows us to calculate approximate values of the desired function $Q(r,t)$ at the nodal points of the difference grid, including nodes on the outer boundary of the reservoir, with the exception of unmarked nodes in the D region.

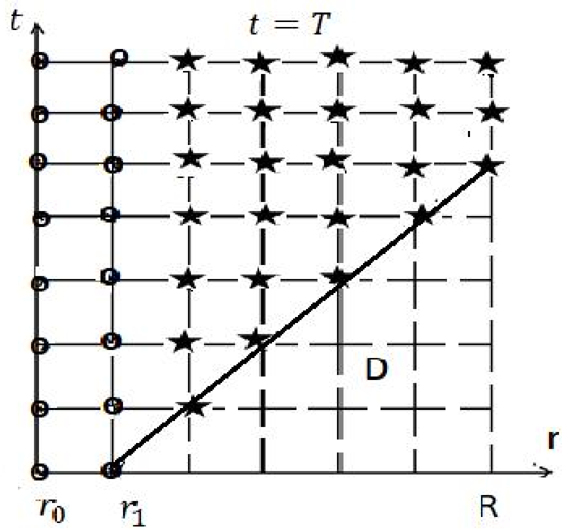

Fig. Discrete domain of numerical solution of the inverse problem

According to the value of the fluid flow rate Q_n^j, $j=\overline{n-1,m}$, at the outer boundary, the presence or absence of a hydrodynamic connection between the reservoir and the outer region is determined.

3 Results of numerical experiments

Based on the proposed computational algorithm, numerical experiments were conducted for a model single-well oil reservoir with the following characteristics:
reservoir radius R =100 m;
layer thickness h =10 m;
the coefficient of absolute permeability of the formation $k = 5\cdot 10^{-12}$ m^2;
well flow rate $Q_w(t)$ = 120 m^3/day;
dynamic viscosity of oil $\mu = 3\cdot 10^{-3}$ Pa $\cdot$ s;
turbulence coefficient β =10^6 m^{-1};
oil density ρ = 850 kg/m^3;
the coefficient of elastic capacity of the reservoir $\beta^* = 2\cdot 10^{-9}$ Pa^{-1}.

Numerical experiments were carried out according to the following scheme:
I. The distribution of the volume flow of liquid in the reservoir at the initial moment of time, the flow rate of the well $Q_w(t)$ and the volume flow of liquid at the outer boundary of the reservoir $Q_R(t)$ are set. A numerical solution of equation (5) satisfying a given initial condition and boundary conditions

$$Q(r_w,t) = Q_w(t)\,, \qquad Q(R,t) = Q_R(t)$$

is determined. In other words, a direct problem is solved numerically to determine the values of the function $Q(r,t)$ at the nodal points of the difference grid $\overline{\omega}$.

II. In the course of solving the direct problem, a variable $\dfrac{df(t)}{dt}$ is determined from relation (8) and the resulting dependence $\dfrac{\partial Q(r_w,t)}{\partial r} = 2\pi r_w h\beta^* \dfrac{df(t)}{dt}$ is taken as an additional condition for solving the inverse problem. (5), (6), (8) to restore the volume flow of liquid at the outer boundary of the reservoir.

According to the proposed scheme, numerical experiments were carried out to restore two functions $Q_R(t) = \dfrac{0.02}{\sqrt{t}}$; $Q_R(t) = 0.0$. At the same time, the self-regularization method (natural regularization method) is used to find a stable solution to the inverse problem [5, 6]. The time discretization step Δt is chosen as the regularization parameter. The calculations were performed on a uniform space-time difference grid with steps of $\Delta r = 3.996$ m, $\Delta t = 30$; 60; 120 s. Numerical experiments revealed that with small sampling steps in time ($\Delta t < 30$ s), an unstable solution appears and the volumetric flow rate of the liquid at the outer boundary of the reservoir is determined by a very large error. However, as the sampling step ($\Delta t \geq 30$ s) increases, the accuracy of reconstructing the volumetric flow rate of the liquid at the boundary increases. The results of numerical experiments conducted at $\Delta t = 60$ s are presented in the table. It contains t –the time, r – the distance from the center of the well, the exact $\overline{Q}$ and calculated values $\widetilde{Q}$ of the function $Q(r,t)$. The dynamics of the distribution of the volume flow of liquid in the reservoir, when the above functions are restored, are presented in columns 2-5 and 6-9, respectively.

Table. Calculated distributions of fluid flow in the reservoir

r, m	$\overline{Q}\times10^3$, $\widetilde{Q}\times10^3$ m³/s				$\overline{Q}\times10^3$, $\widetilde{Q}\times10^3$ m³/s			
	t=10	t=30	t=50	t=60	t=10	t=30	t=50	t=60
0.100	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39
4.096	1.36	1.38	1.39	1.39	1.36	1.38	1.38	1.38
8.092	1.32	1.37	1.38	1.38	1.32	1.37	1.37	1.37
12.088	1.25	1.35	1.37	1.37	1.25	1.34	1.36	1.36
16.084	1.17	1.33	1.35	1.36	1.17	1.31	1.34	1.34
20.080	1.08	1.31	1.34	1.34	1.07	1.28	1.31	1.32
24.076		1.27	1.32	1.32		1.24	1.28	1.29
28.072		1.24	1.29	1.30		1.19	1.25	1.26
32.068		1.20	1.26	1.27		1.14	1.21	1.22
36.064		1.16	1.23	1.24		1.08	1.17	1.18
40.060		1.12	1.20	1.21		1.02	1.12	1.14
44.056		1.08	1.17	1.17		0.96	1.07	1.09
48.052		1.03	1.13	1.13		0.90	1.01	1.03
52.048		0.99	1.09	1.09		0.83	0.96	0.98

56.044		0.95	1.04	1.05		0.76	0.89	0.92
60.040		0.91	0.99	1.00		0.70	0.83	0.85
64.036			0.95	0.95			0.76	0.78
68.032			0.89	0.89			0.69	0.71
72.028			0.84	0.84			0.61	0.64
76.024			0.78	0.78			0.54	0.56
80.020			0.72	0.71			0.46	0.47
84.016			0.66	0.64			0.37	0.39
88.012			0.59	0.57			0.24	0.30
92.008			0.52	0.50			0.19	0.20
96.004			0.44	0.42			0.10	0.10
100.00			0.37	0.33			0.00	0.00

The results of numerical experiments show that with the beginning of reservoir development, the distribution of the volume flow rate of the liquid is restored in a time-varying region. For example, 30 minutes after the start of reservoir development, the distribution of fluid flow is restored within a radius of 60.04 meters around the well. And 50 minutes after the start of development, the fluid flow distribution is fully restored in the reservoir.

Analysis of the results obtained shows that in both variants, the volumetric flow rate of the liquid at the outer boundary of the formation, as well as the dynamics of the distribution of the volumetric flow rate of the liquid in the formation, with the exception of the specified area D, is restored absolutely accurately. Therefore, the exact and calculated distribution of fluid flow in the reservoir are presented in the same column of the table.

4 Conclusion

The problem of determining the conditions at the outer boundary of a single-well oil reservoir developed in an elastic regime is considered. The mathematical model of this problem, based on the law of turbulent filtration, is presented as a boundary-value inverse problem without initial conditions for a nonlinear parabolic equation with respect to the volume flow of a liquid. The proposed computational algorithm allows, based only on the information received from the well, to determine the presence or absence of a hydrodynamic connection between the reservoir and the surrounding area, as well as the dynamics of the distribution of fluid flow during the development of the reservoir in an elastic mode.

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