Synthesis of Terminal Synergetic Controller Based on Nonlinear State Observer for 2-DOF Helicopter System

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Abstract. This paper presents the synthesis of a robust control scheme for a Two Degree-of-Freedom Helicopter System based on Terminal Synergetic Control combined with a Nonlinear State Observer. The 2DOFHS is a highly nonlinear, strongly coupled MIMO system characterized by parametric uncertainties and unknown external disturbances, which makes control design particularly challenging. To address these issues, a terminal synergetic control strategy is developed to ensure finite-time convergence of the pitch and yaw angles to their desired reference values, while guaranteeing closed-loop stability in the presence of bounded disturbances. Since the terminal synergetic control law requires full state information, a nonlinear state observer is designed to estimate the unmeasured state variables, namely the angular velocities of the pitch and yaw motions. The observer is constructed to achieve fast and accurate state estimation, enabling effective implementation of the proposed control law without direct measurement of all system states. Lyapunov-based stability analysis is carried out to rigorously prove finite-time convergence of the tracking errors and asymptotic convergence of the observer estimation errors. The effectiveness and robustness of the proposed controller–observer structure are validated through comprehensive simulation studies under different operating conditions, including set-point regulation and trajectory tracking scenarios, as well as in the presence of bounded external disturbances. The simulation results demonstrate fast transient response, high tracking accuracy, and strong disturbance rejection capability, confirming the superiority of the proposed approach for controlling nonlinear helicopter systems.

Keywords: 2-DOF Helicopter System, Terminal Synergetic Control, Nonlinear State Observer, Lyapunov function.

1 Introduction

The Two Degree-of-Freedom Helicopter System (2DOFHS), also known as the twin-rotor MIMO system, has been widely adopted as a benchmark platform for validating advanced control algorithms in both academic research and practical laboratory environments. This system exhibits several challenging characteristics that make control design nontrivial: (i) it is a highly nonlinear and strongly coupled multiple-input multiple-output (MIMO) system; (ii) it contains parametric uncertainties arising from modeling simplifications and physical variations; (iii) it is subject to unknown external disturbances during operation; and (iv) it requires fast response and guaranteed stability around equilibrium points. Due to these properties, the 2DOFHS represents a realistic testbed for evaluating robustness, performance, and adaptability of modern control strategies.

In early studies, classical control approaches such as PID and linear quadratic regulator (LQR) control were applied to the linearized model of the 2DOFHS under relatively simple disturbance assumptions [1-4]. Although these methods are straightforward to implement and provide acceptable performance near operating points, they generally fail to account for the intrinsic nonlinear dynamics of the system and may suffer from degraded performance when subjected to large disturbances or parameter variations. Consequently, these linear control techniques are insufficient for achieving high-performance control over a wide operating range.

To overcome these limitations, various nonlinear and intelligent control methods have been proposed. Fuzzy logic control schemes were investigated in [5,6] to handle system nonlinearities and uncertainties. Sliding mode control (SMC) approaches were developed in [7-10] to enhance robustness against disturbances, while backstepping-based controllers were introduced in [11,12] to systematically stabilize the nonlinear dynamics. Furthermore, neural network-based control strategies were employed in [13,14] to approximate unknown nonlinearities. Although these methods significantly improve robustness and tracking performance, they often suffer from drawbacks such as chattering, high computational complexity, or sensitivity to tuning parameters.

In recent years, hybrid and adaptive control strategies have gained increasing attention in order to further enhance control performance. Adaptive backstepping control [15-17], adaptive neural control [18,19], and backstepping sliding mode control [20] have been proposed to address uncertainties and improve transient response. Despite their effectiveness, a common assumption in these approaches is the availability of full state measurements. In practical applications, however, not all state variables of the 2DOFHS-particularly angular velocities-are directly measurable, which necessitates the use of state observers.

Synergetic control theory, originally introduced by Kolesnikov [21], has emerged as an effective nonlinear control framework that ensures system stability through the design of invariant manifolds. Unlike sliding mode control, synergetic control eliminates chattering while retaining robustness properties. This control methodology has been successfully applied to a variety of nonlinear systems, including electromechanical systems and power converters [22-25]. More recently, Terminal Synergetic Control (TSC) has been developed as an extension of synergetic control, where terminal dynamics are employed to guarantee finite-time convergence of system states to desired equilibrium points or trajectories [26-29]. Finite-time convergence is particularly desirable in high-performance control applications, as it ensures fast response and improved tracking accuracy.

However, both synergetic and terminal synergetic control laws require accurate knowledge of the system states. To address this issue, several observer-based approaches for the 2DOFHS have been proposed, including sliding mode observers, nonlinear observers, and disturbance observers [30-33]. While these observers can estimate unmeasured states, their integration with terminal synergetic control has not been sufficiently explored.

Motivated by the above observations, this paper proposes a novel control framework that combines Terminal Synergetic Control with a Nonlinear State Observer for the 2DOF Helicopter System in the presence of unknown bounded external disturbances. The nonlinear state observer is designed to estimate the unmeasured angular velocities, enabling full-state feedback implementation of the terminal synergetic controller. Lyapunov-based stability analysis is employed to rigorously prove finite-time convergence of the tracking errors and asymptotic convergence of the observer estimation errors. Comprehensive simulation studies are conducted to demonstrate the effectiveness, robustness, and fast convergence properties of the proposed approach under both set-point regulation and trajectory tracking scenarios.

The remainder of this paper is organized as follows. Section 2 presents the mathematical model of the 2DOF Helicopter System. Section 3 details the design of the terminal synergetic controller and the nonlinear state observer, along with stability proofs. Simulation results are discussed in Section 4 to validate the proposed method. Finally, conclusions and future research directions are provided in Section 5.

2 Mathematical model of 2-DOF Helicopter System

The Two Degree-of-Freedom Helicopter System consists of a rigid beam mounted on a pivot that allows rotational motion in both the vertical (pitch) and horizontal (yaw) planes. Two DC-motor-driven rotors are installed at the opposite ends of the beam: the main rotor generates lift to control the pitch motion, while the tail rotor produces lateral thrust to regulate the yaw motion, as illustrated in Figure 1.

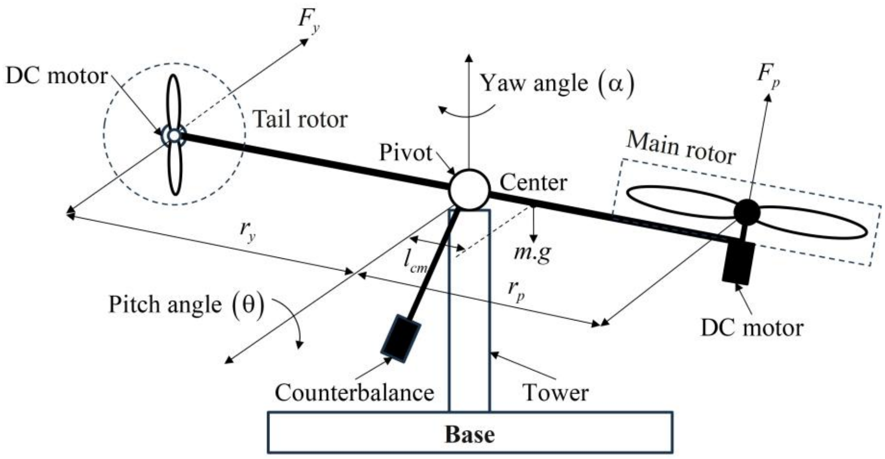

Fig. 1. 2-DOF Helicopter System.

A counterbalance mass is attached near the pivot to establish a stable equilibrium configuration and to compensate for gravitational effects. The entire assembly is fixed on a tower structure, providing a safe and reliable experimental platform for control design and validation. Due to aerodynamic coupling between the rotors, gravitational forces, friction effects, and nonlinear inertial terms, the 2DOFHS exhibits strong non-linear and cross-coupled dynamics. Let θ and α denote the pitch and yaw angles, respectively, while V_p and V_y represent the voltages applied to the main and tail DC motors.

The parameters and their values of TRMS are given in Table 1 [34].

Table 1. Parameters and values of TRMS.

Parameter	Definition	Value
θ	The Pitch angle	-
α	The Yaw angle	-
V_p	The voltage of main DC motor	-
V_y	The voltage of tail DC motor	-
J_p	Moment of inertia about the pitch axis	0.0215 (kgm^2)
J_y	Moment of inertia about the yaw axis	0.0237 (kgm^2)
l_{cm}	Distance between Center of mass and the body-fixed frame origin	0.0071 (m)
m	Weight of the TRMS	1.0750 (kg)
D_p	Pitch viscous friction constant	0.0071 $\left(\frac{N}{V}\right)$
D_y	Yaw viscous friction constant	0.0220 $\left(\frac{N}{V}\right)$
K_{pp}	Torque thrust gain acting on pitch axis from pitch propeller	0.0011 $\left(\frac{Nm}{V}\right)$
K_{py}	Torque thrust gain acting on pitch axis from yaw propeller	0.0021 $\left(\frac{Nm}{V}\right)$
K_{yp}	Torque thrust gain acting on yaw axis from pitch propeller	-0.0027 $\left(\frac{Nm}{V}\right)$
K_{yy}	Torque thrust gain acting on yaw axis from yaw propeller	0.0022 $\left(\frac{Nm}{V}\right)$
g	Gravitational acceleration	9.81 $\left(\frac{m}{s^2}\right)$

Based on Newton-Euler equations, the nonlinear dynamics of the 2DOFHS can be described as follows [8,16]:

$$\begin{aligned}(J_p + ml_{cm}^2)\ddot{\theta} &= K_{pp}V_p + K_{py}V_y - mgl_{cm}\cos\theta - D_p\dot{\theta} - ml_{cm}^2\dot{\alpha}^2 \sin\theta\cos\theta \\ (J_y + ml_{cm}^2\cos^2\theta)\ddot{\alpha} &= K_{yp}V_p + K_{yy}V_y - D_y\dot{\alpha} + 2ml_{cm}^2\dot{\alpha}\dot{\theta}\sin\theta\cos\theta\end{aligned} \tag{1}$$

To simplify the notation and facilitate controller design, the following auxiliary variables are introduced:

$$\begin{aligned}a_1 &= J_p + ml_{cm}^2 \\ a_2 &= -mgl_{cm}\cos\theta - D_p\dot{\theta} - ml_{cm}^2\dot{\alpha}^2\sin\theta\cos\theta \\ b_1 &= J_y + ml_{cm}^2\cos^2\theta\end{aligned}$$

$$b_2 = -D_y\dot{\alpha} + 2ml_{cm}^2\dot{\alpha}\dot{\theta}\, sin\,\theta\, cos\,\theta \quad (2)$$

Using these definitions, the system dynamics can be rewritten in a compact form:

$$\ddot{\theta} = \frac{a_2}{a_1} + \frac{K_{pp}V_p + K_{py}V_y}{a_1}$$
$$\ddot{\alpha} = \frac{b_2}{b_1} + \frac{K_{yp}V_p + K_{yy}V_y}{b_1} \quad (3)$$

To account for unknown external disturbances acting on the system, the state-space representation of the 2DOFHS is formulated as:

$$\begin{cases} \dot{x}_1 = f_1(x) + g_1(x)x_2 \\ \dot{x}_2 = f_2(x) + u_2(t) + d_2(t) \\ \dot{x}_3 = f_3(x) + g_3(x)x_4 \\ \dot{x}_4 = f_4(x) + u_4(t) + d_4(t) \end{cases} \quad (4)$$

where the state variables are defined as $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = \alpha$, $x_4 = \dot{\alpha}$. The system functions and control inputs are given by: $f_1(x) = f_3(x) = 0$, $g_1(x) = g_3(x) = 1$, $f_2(x) = \frac{a_2}{a_1}$, $u_2(t) = \frac{K_{pp}V_p + K_{py}V_y}{a_1}$, $f_4(x) = \frac{b_2}{b_1}$, $u_4(t) = \frac{K_{yp}V_p + K_{yy}V_y}{b_1}$.

The terms $d_2(t)$ and $d_4(t)$ represent unknown but bounded external disturbances satisfying $|d_i(t)| \le D_i$, $i = 2,4$. This state-space formulation highlights the nonlinear and coupled nature of the system and serves as the basis for the subsequent design of the terminal synergetic controller and nonlinear state observer.

3 Design of Terminal Synergetic Controller based on Nonlinear State Observer for 2-DOF Helicopter System

In this section, a terminal synergetic control scheme combined with a nonlinear state observer is developed for the 2-DOF Helicopter System subject to unknown bounded external disturbances. The proposed approach aims to achieve finite-time convergence of the tracking errors while guaranteeing closed-loop stability, even when not all state variables are directly measurable.

3.1 Design of the Terminal Synergetic Controller

According to the terminal synergetic control theory presented in [26-29], the controller design is based on the construction of macro variables that define invariant manifolds governing the closed-loop dynamics. These macro variables are selected such that the system trajectories are driven toward the desired equilibrium or reference trajectories in finite time.

For the position states, the first set of macro variables is defined as

$$\psi_i = x_i - x_{ir}, \quad i = 1,3 \quad (5)$$

where x_{ir} denotes the desired reference trajectory of x_i.

For the velocity states, the second set of macro variables is defined as

$$\psi_i = x_i - \phi_i, \quad i = 2,4 \tag{6}$$

where ϕ_i represents internal virtual control inputs to be designed.

To enforce finite-time convergence, the terminal dynamic evolution of the macro variables is chosen as

$$\psi_i + T_i \dot{\psi}_i^{\frac{p_c}{q_c}} = 0 \tag{7}$$

where $T_i > 0$ for $i = \overline{1,4}$ are design parameters, p_c and q_c are positive odd integers satisfying $1 < \frac{p_c}{q_c} < 2$. This terminal dynamics guarantees that the macro variables converge to zero in finite time.

From (7), the macro-variable dynamics can be expressed as

$$\dot{\psi}_i = W_i \tag{8}$$

where $W_i = \left(-\frac{1}{T_i}\psi_i\right)^{\frac{q_c}{p_c}}$ for $i = \overline{1,4}$.

Substituting the system dynamics into the macro-variable definitions yields

$$\begin{cases} f_1(x) + g_1(x)x_2 - \dot{x}_{1r} = W_1 \\ f_3(x) + g_3(x)x_4 - \dot{x}_{3r} = W_3 \end{cases} \tag{9}$$

On the invariant manifold $\psi_i = 0$, it follows that $x_2 = \phi_2$ and $x_4 = \phi_4$. From (9)

$$\begin{cases} f_1(x) + g_1(x)\phi_2 - \dot{x}_{1r} = W_1 \\ f_3(x) + g_3(x)\phi_4 - \dot{x}_{3r} = W_3 \end{cases} \tag{10}$$

Hence, the internal control laws are obtained as

$$\begin{cases} \phi_2 = g_1^{-1}(x)(W_1 - f_1(x) + \dot{x}_{1r}) \\ \phi_4 = g_3^{-1}(x)(W_3 - f_3(x) + \dot{x}_{3r}) \end{cases} \tag{11}$$

Next, the external control inputs are designed to stabilize the velocity-related macro variables. From the system dynamics, one obtains

$$\begin{cases} f_2(x) + u_2(t) - \dot{\phi}_2 = W_2 \\ f_4(x) + u_4(t) - \dot{\phi}_4 = W_4 \end{cases} \tag{12}$$

Accordingly, the external control laws are given by

$$\begin{cases} u_2(t) = W_2 - f_2(x) + \dot{\phi}_2 + \varphi_2 - D_2 \, sgn \, \psi_2 \\ u_4(t) = W_4 - f_4(x) + \dot{\phi}_4 + \varphi_4 - D_4 \, sgn \, \psi_4 \end{cases} \tag{13}$$

where the additional coupling compensation terms are defined as $\varphi_2 = -g_1(x)\psi_1$ and $\varphi_4 = -g_3(x)\psi_3$.

Theorem 1. The nonlinear system (4) under the control laws (13) is locally stabilized, and the tracking errors converge to zero in finite time T_s, given by

$$T_s = t_0 + \frac{V^{1-\eta}(t_0)}{\beta(1-\eta)} \tag{14}$$

where $V = \frac{1}{2}\sum_{i=1}^{4}\psi_i^2$, $\eta = \frac{p_c+q_c}{2p_c}$, and $\beta = 2^{\eta}\min\limits_{1\le i\le 4}\left(\frac{1}{T_i}\right)$.

Proof. Consider the Lyapunov candidate function

$$V = \frac{1}{2}\psi_1^2 + \frac{1}{2}\psi_2^2 + \frac{1}{2}\psi_3^2 + \frac{1}{2}\psi_4^2 \tag{15}$$

The derivative of the Lyapunov function is determined as

$$\dot{V} = \psi_1\dot{\psi}_1 + \psi_2\dot{\psi}_2 + \psi_3\dot{\psi}_3 + \psi_4\dot{\psi}_4 \tag{16}$$

Then

$$\begin{aligned}\dot{V} &= \psi_1(\dot{x}_1 - \dot{x}_{1r}) + \psi_2(\dot{x}_2 - \dot{\phi}_2) + \psi_3(\dot{x}_3 - \dot{x}_{3r}) + \psi_4(\dot{x}_4 - \dot{\phi}_4)\\ &= \psi_1(f_1(x) + g_1(x)x_2 - \dot{x}_{1r}) + \psi_2(f_2(x) + u_2(t) + d_2(t) - \dot{\phi}_2)\\ &\quad + \psi_3(f_3(x) + g_3(x)x_4 - \dot{x}_{3r}) + \psi_4(f_4(x) + u_4(t) + d_4(t) - \dot{\phi}_4)\\ &= \psi_1(f_1(x) + g_1(x)(\psi_2 + \phi_2) - \dot{x}_{1r}) + \psi_2(f_2(x) + u_2(t) + d_2(t) - \dot{\phi}_2)\\ &\quad + \psi_3(f_3(x) + g_3(x)(\psi_4 + \phi_4) - \dot{x}_{3r}) + \psi_4(f_4(x) + u_4(t) + d_4(t) - \dot{\phi}_4)\\ &= \psi_1(f_1(x) + g_1(x)\psi_2 + g_1(x)\phi_2 - \dot{x}_{1r}) + \psi_2(f_2(x) + u_2(t) + d_2(t) - \dot{\phi}_2)\\ &\quad + \psi_3(f_3(x) + g_3(x)\psi_4 + g_3(x)\phi_4 - \dot{x}_{3r}) + \psi_4(f_4(x) + u_4(t) + d_4(t) - \dot{\phi}_4)\end{aligned} \tag{17}$$

Substituting the internal controls (11) and external controls (13) into (17) yields

$$\begin{aligned}\dot{V} &= \psi_1(f_1(x) + g_1(x)\phi_2 - \dot{x}_{1r} + g_1(x)\psi_2)\\ &\quad + \psi_2(f_2(x) + W_2 - f_2(x) + \dot{\phi}_2 + \varphi_2 - D_2\, sgn\, \psi_2 + d_2(t) - \dot{\phi}_2)\\ &\quad + \psi_3(f_3(x) + g_3(x)\phi_4 - \dot{x}_{3r} + g_3(x)\psi_4)\\ &\quad + \psi_4(f_4(x) + W_4 - f_4(x) + \dot{\phi}_4 + \varphi_4 - D_4\, sgn\, \psi_4 + d_4(t) - \dot{\phi}_4)\\ &= \psi_1(W_1 + g_1(x)\psi_2) + \psi_2(W_2 + \varphi_2 - D_2\, sgn\, \psi_2 + d_2(t))\\ &\quad + \psi_3(W_3 + g_3(x)\psi_4) + \psi_4(W_4 + \varphi_4 - D_4\, sgn\, \psi_4 + d_4(t))\\ &= \psi_1 W_1 + \psi_1 g_1(x)\psi_2 + \psi_2 W_2 + \psi_2\varphi_2 - \psi_2(D_2\, sgn\, \psi_2 - d_2(t))\\ &\quad + \psi_3 W_3 + \psi_3 g_3(x)\psi_4 + \psi_4 W_4 + \psi_4\varphi_4 - \psi_4(D_4\, sgn\, \psi_4 - d_4(t))\end{aligned} \tag{18}$$

After substituting additional terms into (18) and note that $|d_i(t)| \le D_i$, $i = 2{,}4$:

$$\begin{aligned}\dot{V} &= \psi_1 W_1 + \psi_1 g_1(x)\psi_2 + \psi_2 W_2 + \psi_2(-g_1(x)\psi_1) - \psi_2(D_2\, sgn\, \psi_2 - d_2(t))\\ &\quad + \psi_3 W_3 + \psi_3 g_3(x)\psi_4 + \psi_4 W_4 + \psi_4(-g_3(x)\psi_3) - \psi_4(D_4\, sgn\, \psi_4 - d_4(t))\\ &\le \psi_1 W_1 + \psi_2 W_2 + \psi_3 W_3 + \psi_4 W_4 - |\psi_2|(D_2 - d_2(t)) - |\psi_4|(D_4 - d_4(t))\\ &\le \psi_1 W_1 + \psi_2 W_2 + \psi_3 W_3 + \psi_4 W_4\end{aligned} \tag{19}$$

Because p_c and q_c are positive odd integers. Therefore,

$$\begin{aligned}\dot{V} &\le \psi_1\left(-\frac{1}{T_1}\psi_1\right)^{\frac{q_c}{p_c}} + \psi_2\left(-\frac{1}{T_2}\psi_2\right)^{\frac{q_c}{p_c}} + \psi_3\left(-\frac{1}{T_3}\psi_3\right)^{\frac{q_c}{p_c}} + \psi_4\left(-\frac{1}{T_4}\psi_4\right)^{\frac{q_c}{p_c}} \\ &= (-1)^{\frac{q_c}{p_c}}\left(\frac{1}{T_1}\right)^{\frac{q_c}{p_c}}(\psi_1)^{\frac{q_c}{p_c}+1} + (-1)^{\frac{q_c}{p_c}}\left(\frac{1}{T_2}\right)^{\frac{q_c}{p_c}}(\psi_2)^{\frac{q_c}{p_c}+1} \\ &+ (-1)^{\frac{q_c}{p_c}}\left(\frac{1}{T_3}\right)^{\frac{q_c}{p_c}}(\psi_3)^{\frac{q_c}{p_c}+1} + (-1)^{\frac{q_c}{p_c}}\left(\frac{1}{T_4}\right)^{\frac{q_c}{p_c}}(\psi_4)^{\frac{q_c}{p_c}+1} \\ &= -\left(\frac{1}{T_1}\right)^{\frac{q_c}{p_c}}(\psi_1)^{\frac{q_c}{p_c}+1} - \left(\frac{1}{T_2}\right)^{\frac{q_c}{p_c}}(\psi_2)^{\frac{q_c}{p_c}+1} \\ &\qquad - \left(\frac{1}{T_3}\right)^{\frac{q_c}{p_c}}(\psi_3)^{\frac{q_c}{p_c}+1} - \left(\frac{1}{T_4}\right)^{\frac{q_c}{p_c}}(\psi_4)^{\frac{q_c}{p_c}+1}\end{aligned} \tag{20}$$

Let $\lambda_i = \left(\frac{1}{T_i}\right)^{\frac{q_c}{p_c}}$, $i = \overline{1,4}$. We have

$$\begin{aligned}\dot{V} &\le -min\{\lambda_1,\lambda_2,\lambda_3,\lambda_4\}\left((\psi_1)^{\frac{q_c}{p_c}+1} + (\psi_2)^{\frac{q_c}{p_c}+1} + (\psi_3)^{\frac{q_c}{p_c}+1} + (\psi_4)^{\frac{q_c}{p_c}+1}\right) \\ &= -\lambda\left(2^{\frac{q_c+p_c}{2p_c}}\right)\left(\left(\frac{\psi_1^2}{2}\right)^{\frac{q_c+p_c}{2p_c}} + \left(\frac{\psi_2^2}{2}\right)^{\frac{q_c+p_c}{2p_c}} + \left(\frac{\psi_3^2}{2}\right)^{\frac{q_c+p_c}{2p_c}} + \left(\frac{\psi_4^2}{2}\right)^{\frac{q_c+p_c}{2p_c}}\right)_{min} \\ &= -\beta\left(\left(\frac{\psi_1^2}{2}\right)^{\eta} + \left(\frac{\psi_2^2}{2}\right)^{\eta} + \left(\frac{\psi_3^2}{2}\right)^{\eta} + \left(\frac{\psi_4^2}{2}\right)^{\eta}\right)\end{aligned} \tag{21}$$

where $\lambda\, min\{\lambda_1,\lambda_2,\lambda_3,\lambda_4\}_{min}$, $\beta = \lambda\left(2^{\frac{q_c+p_c}{2p_c}}\right)_{min}$, $\eta = \frac{q_c+p_c}{2p_c}$.

So that,

$$\dot{V} \le -\beta\left(\frac{\psi_1^2}{2} + \frac{\psi_2^2}{2} + \frac{\psi_3^2}{2} + \frac{\psi_4^2}{2}\right)^{\eta} = -\beta V^{\eta} \tag{22}$$

The inequality (22) implies that the settling time T_s is determined as (14) [29].

From the definition of $u_2(t)$ and $u_4(t)$, the supplied voltages of DC motors are found

$$\begin{cases} V_p = \dfrac{(J_y + ml_{cm}^2 \cos^2 x_1)u_4(t) - \frac{K_{yy}}{K_{py}}(J_p + ml_{cm}^2)u_2(t)}{K_{yp} - \frac{K_{yy}K_{pp}}{K_{py}}} \\ V_y = \dfrac{(J_p + ml_{cm}^2)u_2(t) - K_{pp}V_p}{K_{py}} \end{cases} \tag{23}$$

■

3.2 Design of Nonlinear State Observer

In practical implementations, the angular velocities of the pitch and yaw motions are often not directly measurable. Therefore, a nonlinear state observer is designed to estimate these unmeasured states.

By grouping the system states, the dynamics can be rewritten as

$$\begin{cases} \dot{x}_{13} = x_{24}, \\ \dot{x}_{13} = f(x_{24}) + g(x_{13})u(t) + d(t) \end{cases} \tag{24}$$

where $x_{13} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$, $x_{24} = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}$, and $d = \begin{bmatrix} d_2(t) \\ d_4(t) \end{bmatrix}$.

Let $\hat{x}_{13} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_3 \end{bmatrix}$ and $\hat{x}_{24} = \begin{bmatrix} \hat{x}_2 \\ \hat{x}_4 \end{bmatrix}$ denote the estimated state vectors of x_{13} and x_{24}, respectively. The proposed nonlinear state observer is formulated as

$$\begin{cases} \dot{\hat{x}}_{13} = \hat{x}_{24} + K\mathrm{sgn}(x_{13} - \hat{x}_{13}) \\ \dot{\hat{x}}_{24} = f(\hat{x}_{24}) + g(\hat{x}_{13})u + LK\mathrm{sgn}(x_{13} - \hat{x}_{13}) \end{cases} \tag{25}$$

where $u = \begin{bmatrix} V_p \\ V_y \end{bmatrix}$, $K = \begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{bmatrix}$ and $L = \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix}$ are positive definite observer gain matrices.

Assume $\tilde{x}_{13} = x_{13} - \hat{x}_{13}$ and $\tilde{x}_{24} = x_{24} - \hat{x}_{24}$. Then, the estimated error dynamics are expressed as

$$\begin{cases} \dot{\tilde{x}}_{13} = \tilde{x}_{24} - K\, sgn(\tilde{x}_{13}) \\ \dot{\tilde{x}}_{24} = f(x_{24}) - f(\hat{x}_{24}) + \big(g(x_{13}) - g(\hat{x}_{13})\big)u - LK\, sgn(\tilde{x}_{13}) \end{cases} \tag{26}$$

Define $c_2 = \frac{D_p}{a_1}, c_3 = \frac{ml_{cm}^2}{a_1}, c_4 = \frac{D_y}{b_1}$, and $c_5 = \frac{2ml_{cm}^2}{b_1}$, and assume that $c_2 \approx c_3 \approx c_4 \approx c_5 = c$ and $a_1 \approx b_1 = a$.

Proposition 1. Consider the observer (25) and assume that positive gains $\kappa_i,\ l_i$ for $i\ = 1{,}2$ satisfy

$$\begin{cases} \kappa_1 > \left|\dot{\hat{\theta}}\right|, \kappa_2 > \left|\dot{\hat{\alpha}}\right| \\ \left(\left(l_1 + \frac{c}{a}\right)\left(l_2 + \frac{c}{a} + \zeta_1\right) > \zeta_2\right. \end{cases} \tag{27}$$

where $\zeta_1 = \frac{c}{a}\dot{\hat{\theta}}\, cos\,\theta\, sin\,\theta$ and $z_2 = \dfrac{c^2}{4a^2}\dot{\hat{a}}^2 \cos^2 q \sin^2 q$. Then, the estimated error converges to zero.

Proof. Consider the observer error dynamics (26). During $\tilde{x}_1 = 0$ and thus $\tilde{x}_2 = K\, \widetilde{sgn}\, x_1$. So that

$$\dot{\tilde{x}}_2 = -L\tilde{x}_2 - \begin{bmatrix} \frac{c}{a} & 0 \\ 0 & \frac{c}{a} \end{bmatrix}\tilde{x}_2 + cos\,\theta\, sin\,\theta \begin{bmatrix} -\frac{c}{a}\left(\dot{\alpha}^2 - \dot{\hat{\alpha}}^2\right) \\ \frac{c}{a}\left(\dot{\theta}\dot{\alpha} - \dot{\hat{\theta}}\dot{\hat{\alpha}}\right) \end{bmatrix} + \Delta_g u \tag{28}$$

where $\Delta_g = g(x_1) - g(\hat{x}_1) \approx 0$ because Δ_g only depends on x_1.

The Lyapunov function is chosen as

$$V(\tilde{x}_2) = \frac{1}{2}\left(\dot{\tilde{\theta}}^2 + \dot{\tilde{\alpha}}^2\right) \tag{29}$$

From (28) and (29)

$$\dot{V}(\tilde{x}_2) = -\left(l_1 + \frac{c}{a}\right)\dot{\tilde{\theta}}^2 - \left(l_2 + \frac{c}{a}\right)\dot{\tilde{\alpha}}^2 + \frac{c}{a}\cos\theta\sin\theta\left(-\dot{\tilde{\alpha}}\dot{\tilde{\theta}}\dot{\hat{\alpha}} + \dot{\tilde{\alpha}}^2\dot{\hat{\theta}}\right) \tag{30}$$

And after that

$$\dot{V}(\tilde{x}_2) = -\begin{bmatrix}\dot{\tilde{\theta}} \\ \dot{\tilde{\alpha}}\end{bmatrix}^T \begin{bmatrix} l_1 + \frac{c}{a} & -\frac{c}{2a}\dot{\hat{\alpha}}\cos\theta\sin\theta \\ -\frac{c}{2a}\dot{\hat{\alpha}}\cos\theta\sin\theta & l_2 + \frac{c}{a} + \frac{c}{a}\dot{\hat{\theta}}\cos\theta\sin\theta \end{bmatrix}\begin{bmatrix}\dot{\tilde{\theta}} \\ \dot{\tilde{\alpha}}\end{bmatrix} = -\dot{\tilde{x}}_2^T M \dot{\tilde{x}}_2 \tag{31}$$

where $\dot{\tilde{x}}_2 = \begin{bmatrix}\dot{\tilde{\theta}} \\ \dot{\tilde{\alpha}}\end{bmatrix}$ and $M = \begin{bmatrix} l_1 + \frac{c}{a} & -\frac{c}{2a}\dot{\hat{\alpha}}\cos\theta\sin\theta \\ -\frac{c}{2a}\dot{\hat{\alpha}}\cos\theta\sin\theta & l_2 + \frac{c}{a} + \frac{c}{a}\dot{\hat{\theta}}\cos\theta\sin\theta \end{bmatrix}$.

The conditions (27) implies that all the principal minors of M are positive definite, and therefore $\dot{V}(\tilde{x}_2)$ is negative definite. Thus, the estimation error converges to zero at an exponential rate. Hence, the states x_2 are obtained exponentially fast. The observer gains $l_i (i = 1,2)$ are chosen relatively large so that the observer converges much faster than the system dynamics.

■

Combination of Terminal Synergetic Controller and Nonlinear State Observer, the control laws (23) can be written

$$\begin{cases} V_p = \dfrac{(J_y + ml_{cm}^2\cos^2(\hat{x}_1))u_4(t) - \frac{K_{yy}}{K_{py}}(J_p + ml_{cm}^2)u_2(t)}{K_{yp} - \frac{K_{yy}K_{pp}}{K_{py}}} \\ V_y = \dfrac{(J_p + ml_{cm}^2)u_2(t) - K_{pp}V_p}{K_{py}} \end{cases} \tag{32}$$

where the external control laws are $u_2(t) = W_2 - f_2(\hat{x}) + \dot{\phi}_2 + \varphi_2 - D_2\, sgn\,\psi_2$ and $u_4(t) = W_4 - f_4(\hat{x}) + \dot{\phi}_4 + \varphi_4 - D_4\, sgn\,\psi_4$, the internal control laws are $\phi_2 = g_1^{-1}(\hat{x})(W_1 - f_1(\hat{x}) + \dot{x}_{1r})$ and $\phi_4 = g_3^{-1}(\hat{x})(W_3 - f_3(\hat{x}) + \dot{x}_{3r})$, the additional terms are $\varphi_2 = -g_1(\hat{x})\psi_1$, $\varphi_4 = -g_3(\hat{x})\psi_3$, $D_i = max(|d_i(t)|)$ for $i = 2,4$.

In practical implementation and simulations, the discontinuous sign function $\mathrm{sgn}(\cdot)$ in nonlinear state observers (25), and the synergetic control law (32) is replaced by the function $\mathrm{sat}(\cdot)$ to alleviate chattering without affecting stability.

4 Simulation Results

In all simulation, the controller parameters are selected as $T_1 = T_2 = T_3 = T_4 = 0.08$, $p_c = 17$, $q_c = 15$ and and the observer gains are chosen as $\kappa_1 = \kappa_2 = 2$, $l_1 = l_2 = 40$ while the plant parameters follow Table 1. To assess robustness, the system is excited by bounded random disturbances acting on the acceleration channels $d_2(t)$ and $d_4(t)$, with bounds $D_2 = D_4 = 2$. The initial values of state observer are $\left[\hat{\theta}(0) \quad \dot{\hat{\theta}}(0) \quad \hat{\alpha}(0) \quad \dot{\hat{\alpha}}(0)\right]^T$=$[0 \quad 0 \quad 0 \quad 0]^T$.

To show the effectiveness of the proposed controller (32), the author considered two cases:

Case 1: Set-point regulation (constant references):

In the first scenario, the desired equilibrium is selected as $[\theta_d(t), \alpha_d(t)]^T = [1, 2]^T$ and the system starts from a displaced condition $\left[\theta(0), \dot{\theta}(0), \alpha(0), \dot{\alpha}(0)\right]^T = [2, 2, 3, 3]^T$.

Figures 2, 3 demonstrate that both outputs converge rapidly to their desired set points despite the presence of unknown disturbances, indicating strong disturbance rejection and effective decoupling behavior in the closed loop. Importantly, Figures 4, 5 show that the observer provides fast and accurate estimates: the estimation errors of both angles and angular velocities converge to (near) zero after a short transient, which validates the suitability of the NOS for implementing the full-state TSC law.

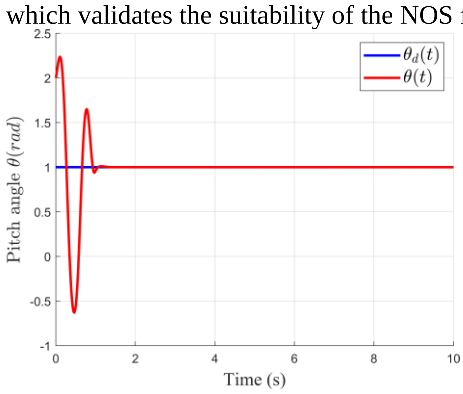

Fig. 2. The Pitch angle of Case 1.

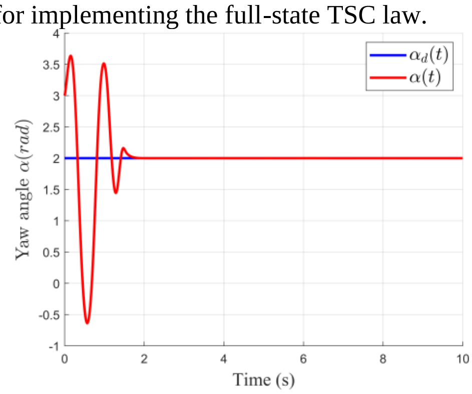

Fig. 3. The Yaw angle of Case 1.

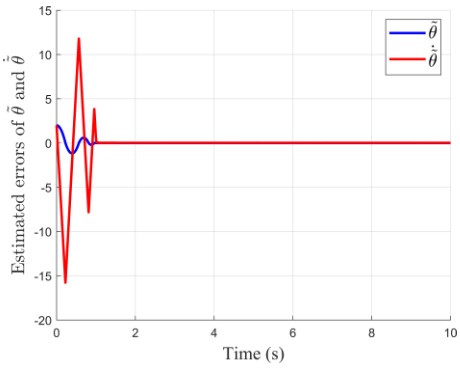

Fig. 4. Estimated errors of Pitch angle and its velocity of Case 1.

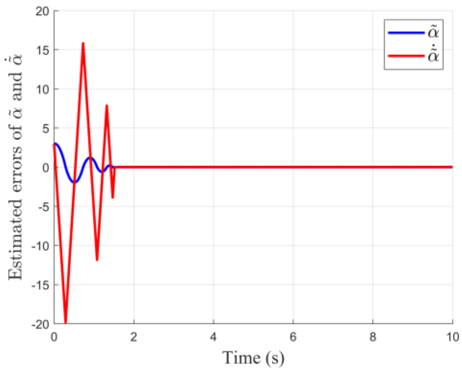

Fig. 5. Estimated errors of Yaw angle and its velocity of Case 1.

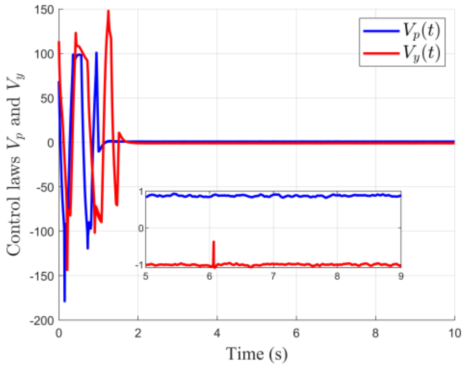

Fig. 6. The control laws of Case 1.

The corresponding control signals (Figure 6) remain bounded and exhibit a short initial transient needed to compensate for the large initial mismatch and disturbances; subsequently, the control effort decreases as the system approaches the invariant manifold and the tracking errors vanish.

Overall, the regulation results confirm that the proposed TSC-NOS structure achieves fast stabilization, high steady-state accuracy, and robustness against bounded perturbations.

Case 2: Trajectory tracking (time-varying references):

To further evaluate tracking capability under time-varying commands, the references are chosen as smooth periodic trajectories: $[\theta_d(t), \alpha_d(t)]^T = [sin\, t\, , cos\, t]^T$ with the same initial state as in Case 1.

Figures 7, 8 show that $\theta(t)$ and $\alpha(t)$ accurately follow the desired trajectories with small tracking error, even during intervals where the reference curvature changes. The estimation-error plots (Figures 9, 10) verify that the observer continues to provide reliable velocity reconstruction in the tracking regime; after the initial transient, the estimated errors remain close to zero, which is critical for maintaining the intended finite-time convergence behavior of the terminal dynamics. The corresponding control inputs (Figure 11) are smooth and bounded, reflecting the inherent advantage of synergetic designs in avoiding the high-frequency switching typically observed in conventional sliding-mode implementations while still preserving robustness.

Discussion: Across both cases, the simulations collectively support the main theoretical claims: (i) the closed loop achieves rapid convergence of tracking errors, (ii) the observer errors decay quickly, enabling practical output-feedback implementation, and (iii) robustness is maintained under bounded disturbances using the compensation terms embedded in the controller design. The presented plots (outputs, estimation errors, and control voltages) therefore provide consistent evidence that the proposed TSC-NOS approach is an effective solution for the highly nonlinear, coupled 2-DOF helicopter dynamics under uncertainty.

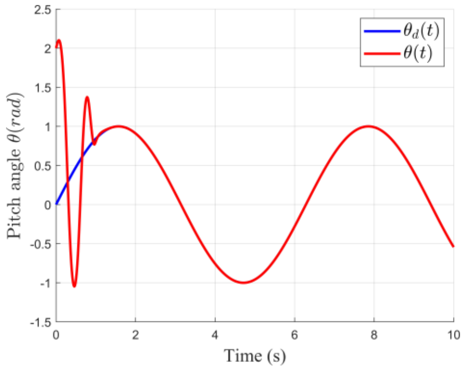

Fig. 7. Pitch angle of Case 2.

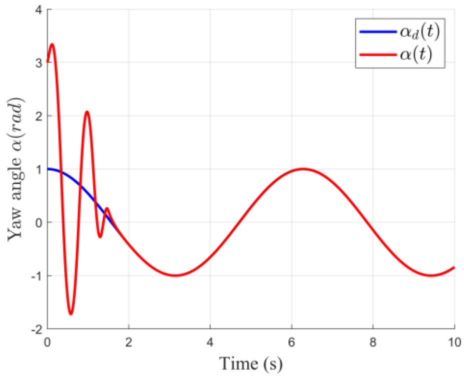

Fig. 8. Yaw angle of Case 2.

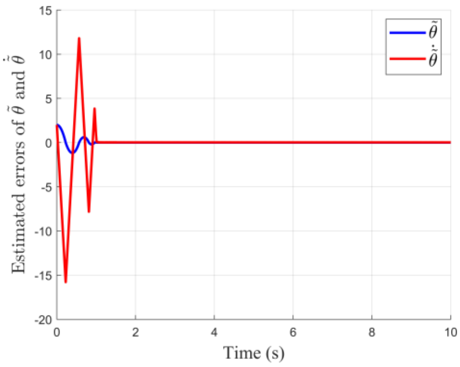

Fig. 9. Estimated errors of Pitch angle and its velocity of Case 2.

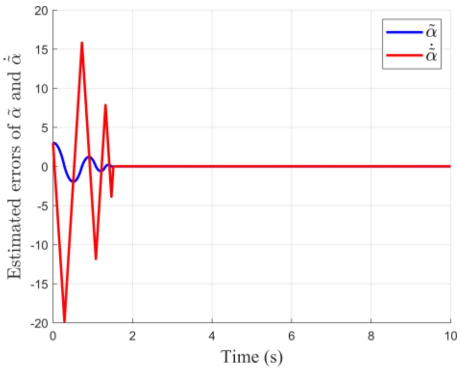

Fig. 10. Estimated errors of Yaw angle and its velocity of Case 2.

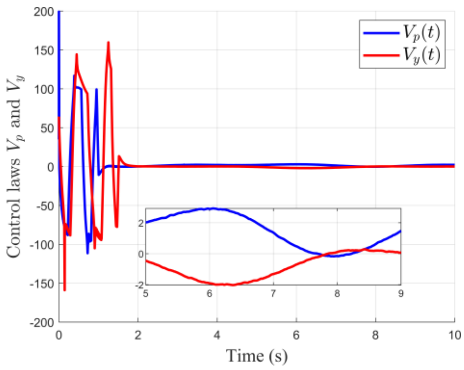

Fig. 11. The control laws of Case 2.

5 Conclusion

This paper has presented a robust observer-based control strategy for the Two Degree-of-Freedom Helicopter System by integrating Terminal Synergetic Control (TSC) with a Nonlinear State Observer. The considered system is characterized by strong nonlinearities, coupling effects, parametric uncertainties, and unknown bounded external disturbances, which pose significant challenges to conventional control techniques.

A terminal synergetic controller was systematically synthesized to ensure finite-time convergence of the pitch and yaw tracking errors to zero. The controller design was carried out using macro-variable construction and terminal dynamics, and its finite-time stability was rigorously established through Lyapunov-based analysis. To address the practical limitation of unavailable velocity measurements, a nonlinear state observer was designed to accurately estimate the angular velocities of the pitch and yaw motions. The convergence of the observer estimation errors was also analytically proven, guaranteeing reliable state reconstruction for output-feedback implementation.

Comprehensive simulation studies were conducted to validate the effectiveness and robustness of the proposed TSC-NOS framework. Both set-point regulation and trajectory-tracking scenarios were examined under unknown bounded disturbances. The results demonstrated fast transient response, high tracking accuracy, and strong disturbance rejection capability. Moreover, the observer exhibited rapid convergence, allowing the controller to maintain the desired finite-time performance without direct access to all state variables. The bounded and smooth control signals further confirm the practical feasibility of the proposed approach.

Overall, the presented results confirm that the proposed terminal synergetic control combined with a nonlinear state observer provides an effective and robust solution for controlling nonlinear helicopter systems with limited state measurements. Compared with existing approaches, the proposed method achieves improved convergence speed and robustness while avoiding excessive control effort.

Future research will focus on extending the proposed framework in several directions. First, the synthesis of an integral terminal synergetic controller will be investigated to further enhance steady-state performance in the presence of persistent disturbances. Second, optimal and adaptive tuning strategies will be explored to automatically select controller and observer parameters, thereby improving performance and reducing manual tuning effort. Finally, experimental validation on a real-time 2-DOF helicopter platform and consideration of actuator saturation constraints will be pursued to further demonstrate the applicability of the proposed method in practical control systems.

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