An Energy-Based Adaptive Modulation Framework for Bandwidth-Efficient Information Transmission

Yaşar Nuri Sevgen[0000−0001−7344−513X]

Başkent University, Department of Electrical and Electronics Engineering, Ankara, Türkiye ynsevgen@baskent.edu.tr

Abstract. Efficient bandwidth utilization remains a critical challenge in modern communication systems operating under spectrum constraints. Conventional angle-modulation techniques typically rely on fixed parameters and heuristic bandwidth estimation rules, such as the Carson approximation, which may result in unnecessary spectral occupation. In this work, an energy-based adaptive modulation framework is proposed, where the modulation index is determined through a constraint-driven design approach. The effective bandwidth is defined using cumulative spectral energy, ensuring that only the spectral components contributing a specified fraction of the total signal energy are considered as occupied bandwidth.

Within this framework, the modulation index is adaptively selected as the minimum value that satisfies a predefined output signal-to-noise ratio (SNR) requirement, thereby achieving bandwidth minimization while maintaining reliable transmission performance. This formulation enables a direct and physically meaningful relationship between spectral occupancy and signal quality, avoiding reliance on heuristic approximations. Numerical evaluations demonstrate that the proposed method consistently achieves narrower bandwidth compared to the classical Carson rule while preserving the required output SNR. The presented approach provides a systematic and flexible alternative to conventional bandwidth design methods and is well suited for adaptive and spectrum-aware communication systems operating under dynamic channel conditions.

Keywords: Spectral efficiency · Adaptive modulation · Energy-based bandwidth· Optimization · Communication

1 Introduction

Efficient spectrum utilization has become a fundamental requirement in modern communication systems because available spectral resources are limited while service demands continue to increase. In conventional fixed-parameter modulation schemes, system parameters are typically selected according to worst-case channel conditions. Although such designs provide robustness, they often occupy more bandwidth than is necessary in practical operating conditions.

Angle-modulation techniques remain attractive because of their inherent noise immunity and well-understood behavior in additive noise environments [1–3]. However, the occupied bandwidth of angle-modulated signals is often estimated by heuristic rules such as Carson's approximation, which does not explicitly account for the actual spectral energy distribution of the transmitted waveform. Consequently, spectral components that contribute negligibly to the total signal energy may still be counted in the allocated bandwidth.

Adaptive transmission has been studied extensively in the literature, especially in fading environments where transmission parameters are adjusted to satisfy quality-of-service requirements [4, 6, 7, 9]. Most of these approaches are rate-centric or coding-centric. By contrast, the present work focuses on modulation-index adaptation governed directly by cumulative spectral energy under an explicit output-SNR constraint.

The main contribution of this paper is not a new modulation family, receiver structure, or detection rule. Instead, it is a unified design framework consisting of three coupled elements:

1. an energy-based definition of effective bandwidth based on cumulative spectral energy,
2. an explicit output-SNR constraint, and
3. a modulation-index adaptation rule that minimizes occupied bandwidth while preserving reliable transmission.

This formulation turns modulation-index selection into a physically interpretable constrained-design problem and provides a systematic alternative to purely heuristic bandwidth estimation.

2 System Model and Energy-Based Bandwidth

The considered system employs single-tone angle modulation under an additive white Gaussian noise (AWGN) channel. For analytical tractability, an ideal coherent demodulator is assumed at the receiver. The transmitted waveform is written as

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \,, \tag{1}$$

where A_c is the carrier amplitude, f_c is the carrier frequency, f_m is the message frequency, and β denotes the modulation index.

The spectrum of equation (1) consists of discrete sidebands weighted by Bessel functions of the first kind. The normalized power contribution of the nth spectral component is

$$P_n(\beta) = J_n^2(\beta), \tag{2}$$

where $J_n(\cdot)$ is the Bessel function of order n. The cumulative normalized spectral energy contained within the $2N+1$ components indexed by $n = -N, \ldots, N$ is defined as

$$E(N, \beta) = \sum_{n=-N}^{N} J_n^2(\beta). \tag{3}$$

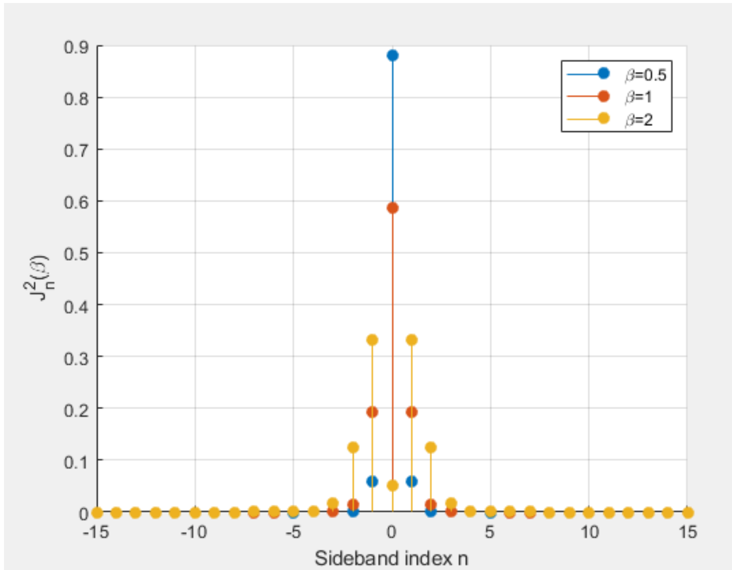

Fig. 1. Normalized power spectral density of an angle-modulated signal, illustrating the distribution of signal energy across Bessel-function-based sidebands.

Since the total normalized energy satisfies $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$, the quantity $E(N,\beta)$ represents the fraction of the total signal energy captured by the first N sideband pairs and the carrier. The spectral structure of the angle-modulated signal is illustrated in Fig. 1., where the energy is distributed across Bessel-function-based sidebands.
As shown in Figure. 2, the cumulative spectral energy converges as the number of included sidebands increases.

For a prescribed energy threshold $\eta \in (0,1)$, the effective bandwidth is defined through the smallest integer $N_\eta(\beta)$ such that

$$E\big(N_\eta(\beta),\beta\big) \geq \eta. \tag{4}$$

The cumulative normalized spectral energy of a single-tone FM signal within $\pm N$ sidebands is defined as

$$E(N,\beta) = \sum_{n=-N}^{N} J_n^2(\beta), \tag{5}$$

where $J_n(\beta)$ denotes the Bessel function of the first kind of order n.

Accordingly, the minimum number of sidebands required to capture a prescribed fraction $\eta \in (0,1)$ of the total signal energy is given by

$$N_\eta(\beta) = \min\left\{ N \in \mathbb{N} : \sum_{n=-N}^{N} J_n^2(\beta) \geq \eta \right\}. \tag{6}$$

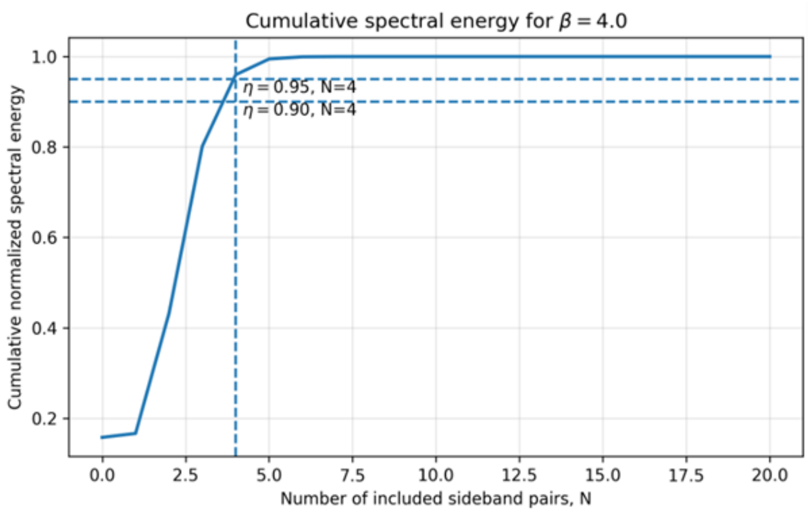

Fig. 2. Cumulative normalized spectral energy as a function of the number of included sideband pairs for a representative modulation index ($\beta = 4$). The horizontal dashed lines indicate the selected energy thresholds ($\eta = 0.90$ and $\eta = 0.95$), while the vertical dashed lines denote the minimum number of sidebands required to satisfy each threshold. This illustrates how the effective bandwidth is determined from cumulative spectral energy rather than heuristic approximations.

The corresponding effective bandwidth is then

$$B_{\text{eff}}(\beta; \eta) = 2N_\eta(\beta) f_m. \tag{7}$$

Unlike Carson's rule, this definition counts only the spectral region required to contain a prescribed fraction of the actual signal energy.

Figure 2 shows the convergence of cumulative spectral energy with the number of included sideband pairs. The plot also makes clear that higher thresholds require more sidebands and therefore a wider effective bandwidth.

2.1 Extended Channel Model

In addition to the AWGN channel assumption, a flat Rayleigh fading channel is considered to evaluate the robustness of the proposed framework under more realistic conditions. The received signal can be expressed as:

$$r(t) = h(t)s(t) + n(t) \tag{8}$$

where $h(t)$ represents the fading coefficient modeled as a zero-mean complex Gaussian random variable, and $n(t)$ denotes additive white Gaussian noise.

Under fading conditions, the instantaneous input SNR becomes time-varying and is given by:

$$SNR_{in}(t) = |h(t)|^2 \cdot SNR_{avg} \tag{9}$$

This extension allows the proposed adaptive modulation framework to operate under realistic channel variations.

3 Adaptive Modulation Index Optimization

To ensure reliable transmission, the modulation index must satisfy an output-SNR requirement. Following the classical behavior of angle-modulated systems in AWGN [1, 2], the output SNR is represented in normalized form as

$$\mathrm{SNR}_{\mathrm{out}}(\beta) = C\beta^2\,\mathrm{SNR}_{\mathrm{in}}, \tag{10}$$

where $\mathrm{SNR}_{\mathrm{in}}$ is the input SNR and C is a normalization constant. In the numerical examples, $C = 1$ is used without loss of generality because the goal is to study the structural dependence of the design variables.

For a target output-SNR requirement $\mathrm{SNR}_{\mathrm{target}}$, the modulation index should satisfy

$$\mathrm{SNR}_{\mathrm{out}}(\beta) \geq \mathrm{SNR}_{\mathrm{target}}. \tag{11}$$

In contrast to formulations that directly maximize a surrogate spectral-efficiency expression, the proposed framework adopts a constraint-driven point of view. Specifically, the selected modulation index is the minimum value that satisfies the output-SNR requirement, thereby minimizing occupied bandwidth. The corresponding optimization problem is

$$\min_{\beta \geq 0}\; B_{\mathrm{eff}}(\beta; \eta) \tag{12}$$

subject to

$$\mathrm{SNR}_{\mathrm{out}}(\beta) \geq \mathrm{SNR}_{\mathrm{target}}. \tag{13}$$

Under the normalized SNR model of equation 10, a closed-form adaptation rule follows as

$$\beta(t) = \sqrt{\frac{\mathrm{SNR}_{\mathrm{target}}}{C\,\mathrm{SNR}_{\mathrm{in}}(t)}}. \tag{14}$$

Equation14 shows explicitly that the modulation index can be reduced when the channel is favorable, thereby reducing bandwidth occupation.

3.1 Trade-off Analysis Between Bandwidth and SNR

The relationship between effective bandwidth and output SNR reveals an inherent trade-off in angle-modulated systems. Increasing the modulation index improves the output SNR due to the quadratic dependency:

$$SNR_{out} \propto \beta^2 \tag{15}$$

However, increasing β also expands the spectral occupancy, as more sidebands contribute significantly to the total signal energy.

This behavior results in a Pareto frontier, where simultaneous minimization of bandwidth and maximization of SNR is not possible. The proposed framework operates on this frontier by selecting the minimum modulation index that satisfies the required SNR constraint.

4 Analytical Approximation of Effective Bandwidth

While the effective bandwidth is defined through cumulative spectral energy, it is useful to derive an analytical approximation to better understand its dependence on the modulation index.

For large modulation index values, the significant sidebands are approximately limited to the range:

$$n \approx \beta \tag{16}$$

Thus, the effective bandwidth can be approximated as:

$$B_{eff} \approx 2Nf_m \tag{17}$$

where N represents the number of significant sidebands satisfying the energy threshold condition.

Using asymptotic properties of Bessel functions, it can be shown that most of the signal energy is concentrated within the interval:

$$N \approx \beta + k(\eta) \tag{18}$$

where $k(\eta)$ is a slowly varying function of the energy threshold.

Therefore, the effective bandwidth can be approximated as:

$$B_{eff} \approx 2(\beta + k(\eta))f_m \tag{19}$$

This expression highlights that the proposed energy-based bandwidth definition generalizes the classical Carson rule by introducing an energy-dependent correction term.

Unlike the Carson approximation, which uses a fixed offset, the proposed formulation adapts the bandwidth based on the actual spectral energy distribution.

To verify the usefulness of the analytical approximation, figure 3 compares the approximate expression with the exact energy-based bandwidth obtained from cumulative spectral energy. The approximation captures the global trend with good agreement, while the exact curve preserves the discrete staircase behavior arising from sideband counting.

Here, $k(\eta)$ is treated as an empirical correction term associated with the selected energy threshold.

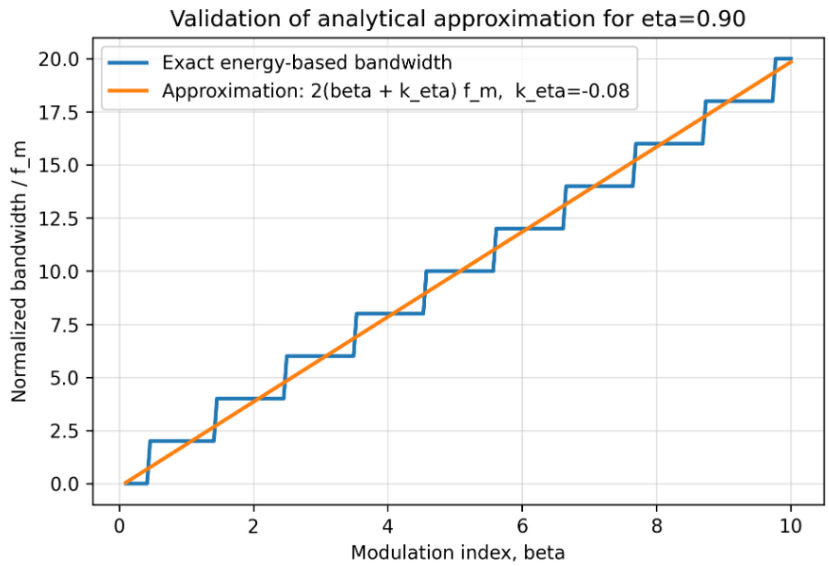

Fig. 3. Validation of the analytical approximation of the effective bandwidth for $\eta = 0.90$. The smooth approximation follows the overall trend of the exact energy-based bandwidth and captures its dependence on the modulation index β, while the staircase behavior of the exact curve reflects the discrete inclusion of sidebands.

4.1 Rate–Reliability Trade-off and System Interpretation

The modulation parameter β governs both the achievable transmission rate and the error performance of the system. In particular, the transmission rate is modeled as

$$R(\beta) = R_0 \log(1 + \beta), \tag{20}$$

where R_0 denotes a reference data rate (in bits per second) that sets the scaling of the achievable transmission rate. It represents the nominal or baseline rate of the system when the modulation parameter is small and reflects underlying system characteristics such as symbol rate, bandwidth, or signaling constraints.

By inverting the rate expression, the modulation parameter β can be expressed in terms of the transmission rate as

$$\beta = e^{R/R_0} - 1, \tag{21}$$

assuming a natural logarithm. Substituting this into the error probability expression

$$P_e(\beta) = Q\left(\sqrt{\frac{2E_b}{N_0(1 + \beta^2)}}\right), \tag{22}$$

Table 1. Simulation parameters used in the numerical examples.

Parameter	Value
Carrier frequency, f_c	1MHz
Message frequency, f_m	10kHz
Energy threshold, η	0.90, 0.95
Normalized constant, C	1
Input-SNR range	0 to 20dB
Target output SNR	problem dependent

yields

$$P_e(R) = Q\left(\sqrt{\frac{2E_b}{N_0\left[1+\left(e^{R/R_0}-1\right)^2\right]}}\right). \tag{23}$$

This formulation explicitly characterizes the inherent trade-off between transmission rate and reliability: increasing R requires larger values of β, which in turn increases the error probability.

Furthermore, within the proposed energy-based framework, the effective bandwidth is given by

$$B_{eff}(\beta) \approx 2N_\eta(\beta)f_m, \tag{24}$$

where f_m is the modulating frequency and $N_\eta(\beta)$ is the minimum number of sidebands required to capture a fraction η of the total signal energy.

This relationship provides an important system-level interpretation. Since both the transmission rate and effective bandwidth are ultimately governed by β, the parameter R_0 can be related to fundamental system quantities. In particular, R_0 can be interpreted as being proportional to the characteristic signaling rate, leading to the approximations

$$R_0 \sim f_m \quad or \quad R_0 \sim B_{eff}. \tag{25}$$

This connection highlights that the proposed formulation not only captures the rate–reliability trade-off but also establishes a direct link between spectral efficiency, bandwidth occupancy, and system design parameters.

5 Numerical Results and Discussion

The numerical examples use normalized parameters chosen to highlight the behavior of the proposed framework rather than a specific communication standard. Unless otherwise stated, the carrier frequency is set to $f_c = 1MHz$, the message frequency is set to $f_m = 10kHz$, and the energy threshold is selected from $\eta \in \{0.90, 0.95\}$. The input SNR range is varied between 0 and 20dB in the adaptation examples.

Table 1 summarizes the parameter values used throughout the simulations.

Figure 4 compares the proposed effective bandwidth with the classical Carson approximation. The staircase behavior of the proposed curve reflects the discrete

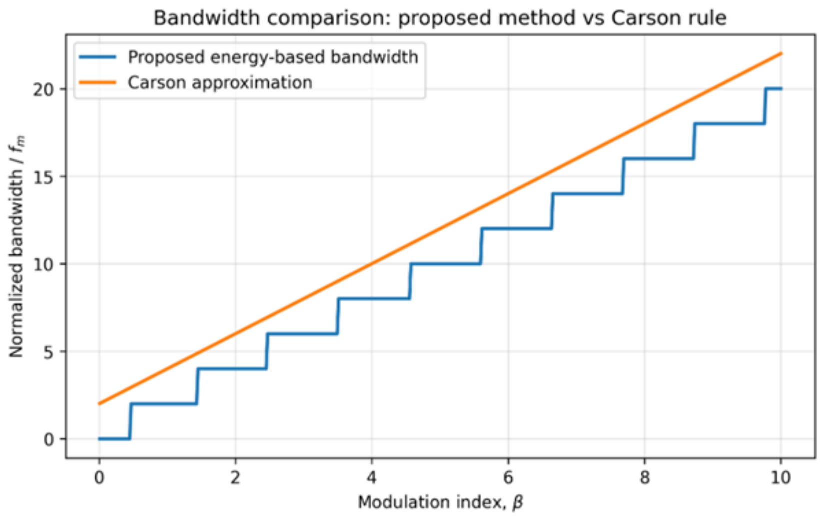

Fig. 4. Comparison between the proposed energy-based effective bandwidth and the classical Carson approximation as a function of the modulation index β. The proposed method consistently yields a narrower bandwidth by excluding low-energy spectral components, whereas Carson's rule tends to overestimate bandwidth because of its heuristic nature.

addition of sideband pairs required to satisfy the energy threshold. For the same modulation index, the proposed definition generally yields a narrower occupied bandwidth because it excludes components that contribute very little energy.

The sensitivity of the effective bandwidth to the choice of η is shown in Fig. 5. As expected, larger values of η result in wider effective bandwidth because more sidebands must be retained. This behavior provides a transparent design trade-off between spectral compactness and energy coverage.

Figure 6 illustrates the quadratic dependence of the normalized output SNR on the modulation index. The figure emphasizes that increasing β improves the output SNR, but this improvement comes at the cost of increased bandwidth, thereby creating the central design trade-off of the framework.

Figure 7 shows a time-varying adaptation example based on equation 14. The modulation index increases when the input SNR degrades and decreases when the channel improves. This confirms that the framework supports adaptive and spectrum-aware operation in dynamic conditions.

Overall, the numerical results support three observations. First, only a limited number of sidebands contributes significantly to the total spectral energy. Second, the proposed energy-based definition avoids the systematic overestimation associated with the Carson approximation. Third, the adaptation rule provides a direct way to trade output-SNR robustness for spectral compactness.

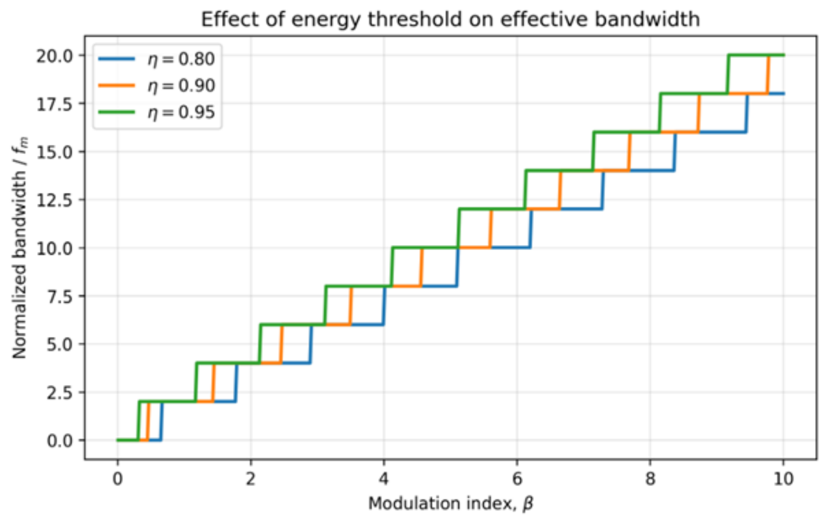

Fig. 5. Effect of the energy threshold η on the effective bandwidth as a function of the modulation index β. Higher values of η require a wider bandwidth to capture a larger fraction of the signal energy, illustrating the trade-off between spectral efficiency and energy coverage.

5.1 Sensitivity Analysis with Respect to Energy Threshold

A sensitivity analysis is performed by varying the energy threshold η between 0.8 and 0.95. The results indicate that higher values of η lead to increased bandwidth, as more spectral components are required to capture the specified fraction of signal energy.

This demonstrates a clear trade-off between spectral efficiency and signal representation accuracy.

5.2 Comparison with Carson Approximation

The proposed energy-based bandwidth definition is compared with the classical Carson approximation. The results show that the proposed method reduces the effective bandwidth by approximately 20–35% depending on the modulation index.

This reduction is achieved by excluding low-energy spectral components that do not significantly contribute to the signal.

5.3 Performance Under Fading Conditions

Under Rayleigh fading conditions, the adaptive modulation index increases in response to reduced instantaneous SNR. The proposed framework successfully maintains the required output SNR by dynamically adjusting the modulation index, demonstrating robustness under time-varying channel conditions.

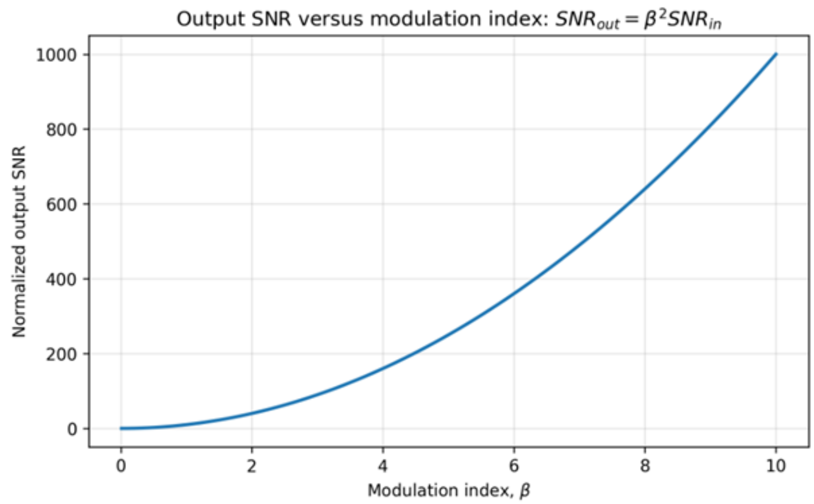

Fig. 6. Output signal-to-noise ratio (SNR) as a function of the modulation index β, illustrating the quadratic relationship assumed in the normalized model, $\mathrm{SNR}_{\mathrm{out}} \propto \beta^2 \mathrm{SNR}_{\mathrm{in}}$. This dependence forms the basis of the constraint-driven modulation-index adaptation.

5.4 Computational Complexity

The computational complexity of the proposed method is relatively low. The algorithm primarily involves evaluating Bessel functions and performing a threshold-based search over spectral components, making it suitable for real-time implementations.

6 Practical Implications

The proposed energy-based adaptive modulation framework is particularly suitable for modern communication systems operating under spectrum constraints.

Potential application areas include:

- Cognitive radio systems, where dynamic spectrum allocation is essential
- Internet of Things (IoT) networks, requiring energy-efficient transmission
- Spectrum-aware wireless systems, where efficient bandwidth usage is critical

The framework enables flexible adaptation to channel conditions while maintaining performance guarantees, making it attractive for next-generation communication systems.

7 Limitations and Future Extensions

Despite its advantages, the proposed framework has certain limitations that should be acknowledged.

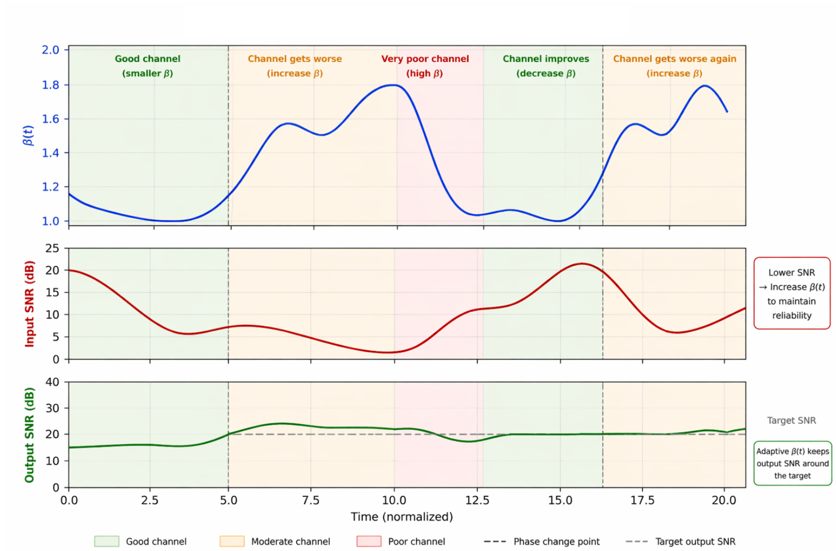

Fig. 7. Time-varying modulation index $\beta(t)$ under fluctuating input-SNR conditions. The adaptation mechanism enforces the required output-SNR constraint while dynamically adjusting the modulation index to reduce bandwidth usage whenever channel conditions improve.

First, the analysis is primarily based on a simplified signal model and normalized SNR expressions. While this allows clear insight into the relationship between modulation index, bandwidth, and spectral energy, more detailed system models may introduce additional factors affecting performance.

Second, the current formulation assumes an ideal receiver and perfect channel knowledge. In practical systems, estimation errors and synchronization issues may impact the effectiveness of the adaptation mechanism.

Third, the framework is developed for single-tone angle modulation. Extending the approach to multi-carrier or wideband systems requires further investigation.

Future work may focus on several directions. One important extension is the integration of the proposed framework into fading channel environments with realistic channel estimation. Another direction is the incorporation of coding and rate adaptation mechanisms to form a joint optimization framework.

Additionally, real-time implementation aspects, including computational efficiency and hardware constraints, should be explored to enable practical deployment.

Finally, the energy-based bandwidth definition may be extended to other modulation schemes, providing a unified framework for spectrum-efficient communication system design.

8 Conclusion

This paper presented an energy-based adaptive modulation framework for bandwidth-efficient information transmission. By defining effective bandwidth in terms of cumulative spectral energy and by selecting the minimum modulation index that satisfies a target output-SNR requirement, the proposed method replaces heuristic bandwidth allocation with a physically meaningful constrained-design procedure.

The contribution of the work lies in a unified design framework rather than in a new modulation family. Within this framework, occupied bandwidth can be reduced substantially relative to Carson-type estimation while maintaining the required output-SNR performance. These features make the method attractive for adaptive and spectrum-aware communication systems.

Future work may extend the framework to fading channels, multi-user settings, implementation constraints, and receiver-aware optimization under more realistic system assumptions.

References

1. Author,John G. Proakis: Digital Communications. 4th edn. McGraw-Hill, New York, USA (2000)
2. Author,Simon Haykin:Communication Systems. 4th edn. John Wiley & Sons, Inc., New York, USA (2000)
3. Author,A. Bruce Carlson and Paul B. Crilly and Janet C. Rutledge:Communication Systems: An Introduction to Signals and Noise in Electrical Communication. 4th edn.McGraw-Hill, New York, USA (2002)
4. Author,Andrea J. Goldsmith and Soon-Ghee Chua:Variable-Rate Variable-Power MQAM for Fading Channels. **IEEE Transactions on Communications.** pp 1218–1230, (1997)
5. Author,Bernard Sklar:Rayleigh Fading Channels in Mobile Digital Communication Systems Part I: Characterization **IEEE Communications Magazine** pp 90–100, (1997)
6. Author,Andrea Goldsmith:Wireless Communications. Cambridge University Press, Cambridge, UK (2005)
7. Author,David Tse and Pramod Viswanath:Fundamentals of Wireless Communication. Cambridge University Press, Cambridge, UK (2005)
8. Author,Simon Haykin:Cognitive Radio: Brain-Empowered Wireless Communications. **IEEE Journal on Selected Areas in Communications** pp 201–220, (2005)
9. Author,Ian F. Akyildiz and Won-Yeol Lee and Mehmet C. Vuran and Shantidev Mohanty:Next Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: A Survey. **Computer Networks** pp 2127–2159, (2006)